Contracting Over Multiple Parameters:
Capacity Allocation in Semiconductor Manufacturing

Suman Mallik
mallik84@opim.wharton.upenn.edu
Corporate Research and Development
General Electric Company
P.O. Box 8
Schenectady, NY 12301

Patrick T. Harker
harker@opim.wharton.upenn.edu
Operations and Information Management
The Wharton School
University of Pennsylvania
Philadelphia, PA 19104-6366

March 1999

Abstract

This paper is a generalization of Mallik & Harker (1998) that presented an integrated model of incentive problems arising in forecasting and capacity allocation. In that model, multiple product managers and multiple manufacturing managers forecast the means of their respective demand and capacity distributions, and a central coordinator allocates capacities based on these forecasts. A mechanism that elicits truthful information from the managers was the main contribution of that paper. The objective of this paper is to generalize our previous results to multiple statistics reporting. This work assumes that the center can ask the managers to report multiple statistics (mean *and* variance, for example) about their respective distributions. We propose a game theoretic model and design a mechanism (a bonus scheme and an allocation rule) that elicits truthful reporting of all statistics by all managers. It turns out that the structure of the optimal bonus schemes are rather simple with easily calculable parameters. We also show that a large class of allocation rules are manipulable. A bonus is often required for elicitation of truthful information. We compare our results of multiple statistics reporting to those from single statistics reporting. We also characterize under what conditions the reporting of the extra information is of limited use.

Keywords: coordination, supply chain management, demand forecasting, game theory, capacity allocations, incentives, mechanism design

1.0 INTRODUCTION

This paper, motivated by the experiences of a major US-based semiconductor manufacturer, presents a generalization of our previous work (Mallik and Harker, 1998). The said firm is in the business of manufacturing and marketing of telecommunications, electronic and computer equipment. The firm operates five wafer fabrication facilities (fabs) and produces 36 major product lines. The total workforce of the firm is about 100,000. On the software side, the firm pursues an aggressive strategy of acquiring the Silicon Valley startup companies. A large proportion of the product mix of the firm is of relatively short life cycle (one or two years). A semiconductor chip loses 60% of its value within first six or seven months of its life cycle. The operating environment of the firm is characterized by volatile demand, high manufacturing leadtime (2-3 months, typically), and rapid change of technology. In order to respond quickly to changing operating environment, the firm has organized itself into a divisionalized structure. A product manager (PM) heads each product group, while a manufacturing manager (MM) heads each wafer fab. Because of the high manufacturing leadtime, production planning and capacity allocation decisions are based on forecasts from the PMs and MMs. Mallik and Harker (1998) (henceforth referred to as MH) studied the incentive problems arising under such circumstances at the said firm.

Since our current work is a generalization of MH, we first provide a brief description of the problem studied in MH. In MH, the demands and capacities of wafer fabs are treated as random and are private knowledge of the respective PMs and MMs. The capacity is a random variable as is the yield in semiconductor manufacturing. Each manager forecasts the mean of his respective demand or capacity distribution. A central coordinator is responsible for allocating capacities to product groups based on these forecasts. In the presence of a capacity shortage, which is often the case, the managers tend to behave strategically and misrepresent their respective forecasts. A PM tends to inflate his forecast with the hope of getting a greater allocation of capacity. A manufacturing manager knows that today's forecast is tomorrow's production quota! Therefore, he tends to be conservative and understates his forecast. MH studied this problem from the perspective of the central coordinator and designed a *mechanism* that induces truthful reporting by all managers. The word *mechanism* is used to mean an allocation rule that the center follows to allocate capacities to different products and a bonus scheme for all managers. In particular, they address the following issues.

- What are the structures of the incentive schemes under different conditions of information availability solution concepts? Are the bonuses always required?
- What kind of allocation rules are manipulable under different conditions of information availability?

To model the fast-changing environment of semiconductor manufacturing, MH considers two conditions of information availability for the central coordinator: the *limited information* (where no prior information is available) and the *partial information* (where prior information is available). They show that the structure of proper bonus function is rather simple with easily calculable parameters. Modified lexicographic allocation was shown to be truthful under partial information.

The MH study only considered the contracting problem over a *single* parameter, namely the mean of a distribution. This paper generalizes the results reported in MH by considering a similar contracting problem over *multiple* parameters. Here we keep the basic structure of the coordination problem identical to that of MH, but assume that the central coordinator can ask the PMs and the MMs to report *multiple* statistics about their respective distributions (for example, mean *and* standard deviation). We propose a game theoretic model and seek to design a mechanism that elicits truthful information about multiple statistics of the unknown distributions. This might be an attractive idea from the perspective of the central coordinator as the extra information might aid the coordination process. Since only the first two moments (or, mean and variance) of a distribution are useful for any practical purpose, we restrict our analyses of multiple statistics reporting to two statistics reporting, namely, the mean and the variance (first two moments of a distribution). However, our results can be generalized to any finite number of statistics reporting. We explicitly point out these generalizations in Section 5. To our knowledge, this paper is the first to report contracting over multiple parameters in the Operations Management literature. We show that it is possible to design a contract that elicits truthful information about multiple parameters. We compare and contrast our current results of multiple statistics reporting with those from MH (single statistics reporting).

The remainder of the paper is organized as follows. The next section reviews the appropriate literature and positions our work with respect to this literature. In Sections 3 and 4, we present a formal game theoretic model that specifically captures the information asymmetry and divergence of preferences of the managers and derive insights from it. Section 5 generalizes our results of two statistics reporting to *n*-statistics reporting. We end with summary and directions for future research in Section 6.

2.0 LITERATURE REVIEW

The incentive issues in intra-firm coordination and supply chain management have been addressed in literature in many different forms and contexts. We provide a brief description of the papers that relate closely to our work. The reader is referred to the work of Tsay, Nahmias, and Agrawal (1998) for a comprehensive review of contracting literature in supply chain management. Whang (1995) provides for a formal classification of coordination problems operations management.

Porteus & Whang (1991) studied a coordination problem between one manufacturing manager and several product managers. The product managers make sales efforts while the manufacturing manager makes efforts for capacity realization and decides inventory levels for different products. All parties act in their self-interest. They seek incentive plans that will induce the managers to act in such a way that owner of the firm can attain maximum possible returns. Grout & Christy (1993) let a buyer give the supplier a bonus for on-time delivery, making the supplier take the buyer's stockout cost into account when determining his stock level. Lee & Whang (1994) study the decentralized version of the multi-echelon inventory model studied by Clark & Scarf (1960). They develop an incentive system that will induce the individual echelon managers to take jointly optimal decisions using locally available information and leaving them at least as well off as they would be under the Clark & Scarf (1960) scheme. Corbett (1996) considered the case of one supplier and one customer where the supplier holds inventory at the customer's premises and the customer has private information about his stockout costs. Cachon & Lariviere (1996) consider the problem of capacity allocation problem of a manufacturer faced with demands from multiple retailers each having private information. They present allocation mechanisms under which truth telling is an equilibrium strategy and conclude that truth telling can lower the profit of the supplier, the supply chain and even the retailers. Cachon & Lariviere (1997) propose a two-period game to model the turn-and-earn allocation observed in the automotive industry.

We study the contracting problem over multiple parameters. To our knowledge, no other paper has studied contracting over multiple parameters in operations management literature. In addition, all of the works described in this section consider *one* manufacturing manager/supplier and multiple customers. In particular, the works by Hauser et al. (1994) and Corbett (1996) involve only one supplier and one customer. In contrast, we consider *multiple* manufacturing managers (suppliers) along with multiple product managers (customers). Our problem, thus, contains a market-like structure which allows us to study the strategic interactions not only between the product managers, but also between the manufacturing managers.

3.0 THE MODEL

The different characteristics of the game theoretic model are specified in this section; the notation used herein is identical to that in MH. The players in the game theoretic model are the individual managers (i.e., the PMs and the MMs). The respective strategies are the reports of the requested statistics that belong to the strategy sets described by the support of the respective random variables. The following subsections formally define the model.

3.1 Notation

i = index on product, $i = 1, 2, ..., N$

k = index on plant, $k = 1, 2, ..., M$

h_i = unit overage cost of product i

p_i = unit shortage cost for product i

e_i = unit revenue for product i

Z_i = random demand for product i, with distribution function $G_i(.)$, density $g_i(.)$, and mean ν_i; z_i is a realization of Z_i

B_k = random capacity for plant k, with distribution function $F_k(.)$, density $f_k(.)$, and mean μ_k; b_k is a realization of B_k

The center requests the expected values of *two* transformations, $T(.) = (T_1(.), T_2(.))$, of the random variables $Z_i(.)$, $i = 1,2, .., N$, and $B_k(.)$, $k = 1,2, \ldots, M$. Thus, $T_1(Z_i) = Z_i$ represents the first moment or the mean, $T_2(Z_i) = Z_i^2$ represents the second moment. Thus, the center requests information about the mean and variance of the unknown distributions (for example, for a PM, mean = EZ_i, and variance = $EZ_i^2 - [EZ_i]^2$).

a_i^j = report of j^{th} transformation from PM $i, j = 1, 2;$

b_k^j = report of j^{th} transformation from MM $k, j = 1, 2;$

$b_k = (b_k^1, b_k^2)$, $b = (b_1, \ldots, b_M)$, $b_{-k} = (b_1, \ldots, b_{k-1}, b_{k+1}, \ldots, b_M)$;

$a_i = (a_i^1, a_i^2)$, $a = (a_1, a_2, \ldots, a_N)$, $a_{-i} = (a_1, a_2, \ldots, a_{i-1}, a_{i+1}, \ldots, a_N)$;

$S_k(\beta, b_k)$ = bonus for MM k;

$r_i(\alpha, z_i)$ = bonus for PM i.

3.2 Assumptions and Timing

The following assumptions will be used throughout the remainder of the paper:

1. Single period model with no inventory carryover.
2. The center as well as all managers are risk neutral.
3. Demands are independent of each other.
4. Capacities are independent of each other and are independent of the demands.
5. Problem parameters (holding, penalty and shortage costs, product revenues etc.) are common knowledge.
6. All managers are treated equally in the allocation process; i.e., personal clout of the managers does not influence the allocation decisions.

The timing of the events is as follows:

1. The managers privately learn the distribution of their respective demands or capacities.
2. The center announces the allocation rule for the product managers and the incentive schemes for both the product and manufacturing managers for all possible realizations of demands and capacities. All managers observe the announcement.

3. The PMs and MMs give their forecasts to the center. Others do not observe the forecast of a manager.
4. Allocations are made by the center and are observed by all parties.
5. Production takes place. Capacities are realized, and are seen by all.
6. Demand is realized and is seen by all. The managers are rewarded as per announced incentive schemes.

3.3 Utility Maximization Problems

The problem faced by MM k, k = 1, 2, …, M, under the assumption of risk neutrality, is given by

$$\underset{b_k^1, b_k^2}{Max}\ E_{F_k} S_k(b, B_k), \tag{1}$$

where $S_k(\beta, B_k)$ is the bonus of MM k. The bonus is a function of the reports of all MMs and the capacity of MM k. Note that this formulation allows us to capture the strategic interactions between the manufacturing managers.

The payoff of PM i, $R_i(a, z_i)$, is the sum of two components: the bonus payment $r_i(.)$ received for participating in the forecasting process; and a *pre-specified* fraction, y_i, of the profit $p_i(.)$ arising from selling the product to the customer. Thus,

$$R_i(a, z_i) = r_i(a, z_i) + y_i\, p_i(\ell_i(a), z_i), y_i \in (0,1), \tag{2}$$

where $\ell_i(a)$ is the allocation rule and denotes the allocation received by product manager i, given the vector of reported forecasts a. We consider a newsboy type profit function for each product manager i, i = 1, 2, ..., N. Thus,

$$p_i(\ell_i(a), Z_i) = e_i \min\{\ell_i(a), Z_i\} - h_i[\ell_i(a) - Z_i]^+ - p_i[Z_i - \ell_i(a)]^+. \tag{3}$$

The problem faced by the PM, under the assumption of risk neutrality, is given by:

$$\underset{a_i^1, a_i^2}{Max}\ E_{G_i} R_i(a, Z_i). \tag{4}$$

The problem faced by the center is to choose the allocation mechanism $\ell_i(\alpha)$ and the incentives $S_k(\beta, b_k)$ and $r_i(\alpha, z_i)$ such that truth telling is the equilibrium strategy for all managers. An optimal allocation maximizes the total profit of the firm less the incentive payments. We have defined the optimal allocation problem in MH in equations (5)-(9). The same definition holds in this paper with the notation $a_i = (a_i^1, a_i^2)$. For easy reference to the reader, we describe the formulation here.

$$\max_{\ell_i, r_i, S_k} E\{\sum_{i=1}^{n}(1-y_i)p_i(\ell_i(a), Z_i) - \sum_{i=1}^{n} r_i(a, Z_i) - \sum_{k=1}^{m} S_k(b, B_k)\} \tag{5}$$

Subject to

$$\sum_{i=1}^{n} \ell_i(a) \le K(b); \forall a, \forall b, \tag{6}$$

$$a_i = \arg\max_{a_i'} E_{G_i} R_i(a_i', a_{-i}; Z_i), \tag{7}$$

$$b_k = \arg\max_{b_k'} E_{F_k} S_k(b_k', b_{-k}; B_k), \tag{8}$$

$$S_k(.)\ge 0, r_i(.)\ge 0, \ell_i(a) \ge 0.. \tag{9}$$

Under an *individually responsive* (IR) allocation mechanism, if one is receiving a positive allocation but wants more, one gets more unless one has already claimed all of capacity. The examples are proportional allocation, linear allocation etc.

3.4 Desirable Properties of Bonus Functions

To discuss the desirable properties of the reward functions $S_k(\beta, b_k)$ and $r_i(\alpha, z_i)$, we restrict our attention to the functions that are piecewise continuously differentiable in their first variable; i.e., $S_k(., .)$ and $r_i(., .)$ are of class $C^{(1)}$ in their first variable except at finitely many points.

1. <u>*Revealing*</u>: A reward function is revealing if all managers truthfully reveal the private information about *each statistics* of their respective distributions. This means that the reward function $S_k(., .)$ must satisfy the following conditions:

$$\frac{\partial}{\partial b_k^1}\{E S_k(b, B_k)\} = 0; \frac{\partial}{\partial b_k^2}\{E S_k(b, B_k)\} = 0, \forall k \tag{10}$$

Similar definition holds for the bonus of the product managers, $r_i(a, Z_i)$.

2. *Accurate*: The definition of accurate is based on outcome, as only the outcome is verifiable. (Note that the random variables Z_i^2 and B_k^2 are never realized). A reward function is accurate if the reward is highest when the prediction exactly equals the outcome and decreases as the deviation between the prediction and outcome increases; i.e., if $S_k(\beta; b_k)$ is decreasing in $|\beta^1_k - b_k|$ and $r_i(\alpha; z_i)$ is decreasing in $|\alpha^1_i - z_i|$. This requirement is assumed to be satisfied if $[\partial S_k / \partial b_k^1]_{b_k^1 = b_k} = 0$ and $S_k(.)$ is *strongly quasiconcave* in b_k^1 for all k; and if $[\partial r_i / \partial a_i^1]_{a_i^1 = z_i} = 0$ and $r_i(.)$ is *strongly quasiconcave* in a_i^1 for all i.

3. *Responsive*: A reward function is responsive if it is more rewarding for a manager to be accurate on a higher forecast than to be accurate on a lower forecast.

4. *Rational*: A reward function is rational if $r_i(\alpha, z_i) = 0$, for $a_i^1 = \underline{z}_i^1, a_i^2 = \underline{z}_i^2$; and $S_k(\beta, B_k) = 0$, for $b_k^1 = \underline{B}_k^1, b_k^2 = \underline{B}_k^2$, where [$\underline{z}_i^1$, $\overline{z_i^1}$] denote the support of the transformation $T_1(Z_i)$ and [$\underline{z}_i^2$, $\overline{z_i^2}$] denote the support of the transformation $T_2(Z_i)$. Similarly, [$\underline{B}_k^1$, $\overline{B_k^1}$] denote the support of the transformation $T_1(B_k)$ and [$\underline{B}_k^2$, $\overline{B_k^2}$] denote the support of the transformation $T_2(B_k)$. If these were not true, then the managers will receive positive reward payments without participating in the forecasting process, which is undesirable.

We call a reward function to be *proper* when it satisfies all the four desirable characteristics mentioned in this sub-section.

3.5 Information Structure

The distributions are private knowledge of the respective managers. Thus, B_k is the private knowledge of MM k, $\forall$k; while Z_i is private knowledge of PM i, $\forall$i. As in MH, we consider two different condition of information availability for the central coordinator:

Limited Information: the demand and capacity distributions are unknown to the central coordinator and no prior information is available, and

Partial Information: the demand and capacity distributions are unknown to the center; however, it does possess a prior over each of the unknown distribution. We consider each of these distributions to be normal with unknown parameters. The motivation for considering these two conditions of information availability is described in MH.

3.6 Solution Concepts

We consider two different solution concepts, the dominant strategy equilibrium and Bayesian Nash equilibrium. We formally define these two concepts in Appendix A.

4.0 ANALYSES

In this section, we seek to characterize the structure of the proper bonus function and the allocation rules that can be implemented by revelation. We also want to study the possibility of implementing an optimal allocation and an IR allocation by revelation under different solution concepts. The dominant strategy equilibrium is studied under the conditions of limited information (Section 4.1) and partial information (Section 4.2). Since the Bayesian equilibrium always assumed the existence of priors, the idea of limited information is no longer relevant in Bayesian equilibrium. The Bayesian equilibrium (under partial information) is studied in Section 4.3. We restrict our analyses to two statistics reporting, namely, the mean and the variance (first two moments of a distribution). This means that $T_1(Z_i) = Z_i$ and $T_2(Z_i) = Z_i^2$. Similarly, $T_1(B_k) = B_k$ and $T_2(B_k) = B_k^2$.

4.1 Dominant Equilibrium Under Limited Information

In this section we assume that the center does not know the distributions of demands or capacities and no prior information is available. The only assumption that we make in this section is that the supports of these random variables are finite and that the center does know all the supports of the random variables. Let $[\underline{Z}_i, \overline{Z}_i],\ -\infty < \underline{Z}_i < \overline{Z}_i < \infty$, be the support of Z_i, for all i, and let $[\underline{B}_k, \overline{B}_k], -\infty < \underline{B}_k < \overline{B}_k < \infty$, be the support of B_k, for all k. Also let $[\underline{z}_i^1, \overline{z}_i^1]$ denote the support of the transformation $T_1(Z_i)$ and let $[\underline{z}_i^2, \overline{z}_i^2]$ denote the support of the transformation $T_2(Z_i)$. Similarly, let $[\underline{B}_k^1, \overline{B}_k^1]$ denote the support of the transformation $T_1(B_k)$ and let $[\underline{B}_k^2, \overline{B}_k^2]$ denote the support of the transformation $T_2(B_k)$. We assume each of the cumulative distribution functions (cdf) of these random variables to be continuously differentiable on their respective domains. The results presented in this section are valid for *any* distribution of demands and capacities. The following lemma forms the basis of analysis for the limited information case. Note that this is a generalization of a similar lemma in our previous analysis of single statistics reporting.

Lemma 1: *Let* $X \in [a,b]$ *be a random variable with distribution function F(.). Let* $T_1(x)$ *and* $T_2(x)$ *be two continuous and strictly monotonic functions of x and* $T(x) = (T_1(x), T_2(x))$. *Also let* $t = (t_1, t_2)$ *denote another vector of finite dimension. A continuously differentiable function g(x): [a, b]* $\rightarrow R$, *satisfies the condition* $\int_a^b g(x)\,dF(x) = 0, \forall F$ *such that* $\int_a^b T(x)\,dF(x) = t$, *if and only if there exists a vector* $k = (k_1, k_2)$, *independent of x, such that*

$g(x) = k_1 [T_1(x) - t_1] + k_2 [T_2(x) - t_2]$.

<u>Proof</u>: All proofs are included in Appendix C.

This lemma tells us that the proper reward function must be linear in outcome. This forms the basis for further analyses. The above lemma gives rise to the following results.

Theorem 1 (Manufacturing Managers): A *bonus function is proper if and only if it is strongly quasiconcave in outcome and is of the following form:*

$$S_k(b, b_k) = u(b_k) - \frac{\partial u(b_k)}{\partial b_k^1}[b_k^1 - T_1(b_k)] - \frac{\partial u(b_k)}{\partial b_k^2}[b_k^2 - T_2(b_k)] + \overline{C}(b_k), \qquad (11)$$

for some appropriate function u(.) and constant $\overline{C}(b_k)$.

Theorem 1 gives the necessary and sufficient condition for a reward function to satisfy all the four desirable characteristics mentioned in Section 3.4. The first term in the above equations represents the expected surplus of a manager and solely depends on the reported information. Hence it can be considered as an ex-ante payment. The manager is penalized (or rewarded) ex-post in proportion to the difference between actual and projected values of the requested statistics. The proportionality factors are given by the gradient of the surplus function. An example of the function *u(.)* that gives rise to a strongly quasiconcave reward function is given by:

$$u(b_k) = (\overline{B_k^1} - b_k^1)[ln(\overline{B_k^1} - b_k^1) - 1] + (\overline{B_k^2} - b_k^2)[ln(\overline{B_k^2} - b_k^2) - 1]. \qquad (12)$$

Substituting (12) into (11) and simplifying we get the following *proper* reward function for MM k:

$$\begin{aligned} S_k(b, b_k) = C_k(b_{-k})\ [\ & (b_k^1 - \underline{B_k^1}) + (\overline{B_k^1} - T_1(b_k))\, ln\frac{\overline{B_k^1} - b_k^1}{\overline{B_k^1} - \underline{B_k^1}} + \\ & (b_k^2 - \underline{B_k^2}) + (\overline{B_k^2} - T_2(b_k))\, ln\frac{\overline{B_k^2} - b_k^2}{\overline{B_k^2} - \underline{B_k^2}} \quad]. \end{aligned} \qquad (13)$$

The bonus function in (13) is a sum of two components that corresponds to the first two moments of the unknown distribution. Each of these two components is sum of two terms. The first term indicates that the reward is linear and increasing with the reported forecast. However, a manager must forecast above $\underline{B_k^1}$ to get a positive reward. The logarithmic term indicates that reward will get lower as the forecast gets closer to $\overline{B_k^1}$. This limits the reward that can be earned through the first term. The coefficient before the log term ensures that the first order condition is satisfied; i.e., that the reward function is accurate. Similar interpretation can be made for the second moment term in (13). Equation (13) tells us that the dominant strategy proper reward is rather simple in structure, linear in outcome and logarithmic in reported forecast. The elegance of this reward structure lies in its easily calculable parameters. The structure of the proper reward function for the single statistics case studied in MH was given by

$$S_k(\beta, b_k) = A_k(\beta_{-k})[(\beta_k - \underline{B_k}) + (\overline{B_k} - b_k)\ln\frac{\overline{B_k} - \beta_k}{\overline{B_k} - \underline{B_k}}]. \qquad (14)$$

Comparing (13) and (14), several interesting observations can be made. Firstly, the proper reward function for the multiple statistics case (13) reduces to that of single statistics case (14) if only the first moment of the unknown distribution is requested from the individual managers. Secondly, the class of proper bonus functions does not seem to "shrink" as more information is elicited. The reason for this is that as the set of reported parameters of a distribution increases, the central coordinator has a wider choice of bonus functions. Thirdly, and most interestingly, the proper bonus function for the multiple statistics case is additive in the requested moments, with a similar functional form in each of the requested moments.

We next look at the product managers. The following theorem describes the structure of proper bonus functions and allocation rules for the product managers.

Theorem 2 (Product Managers): *A reward function $r_i(.)$ and an allocation rule $\ell_i(.)$ for product manager i can be implemented by revelation in dominant equilibrium if $r_i(.)$ and $\ell_i(.)$ satisfy the following relationship for any distribution $G_i(.)$:*

$$\int_{\underline{z}_i}^{\bar{z}_i} \frac{\partial r_i(\alpha, z_i)}{\partial \alpha_i^1} dG_i(z_i) - \psi_i h_i \frac{\partial \ell_i(\alpha)}{\partial \alpha_i^1} G_i[\ell_i(\alpha)] + \psi_i(p_i + \varepsilon_i) \frac{\partial \ell_i(\alpha)}{\partial \alpha_i^1}[1 - G_i(\ell_i(\alpha))] = 0, \tag{15}$$

$$\int_{\underline{z}_i}^{\bar{z}_i} \frac{\partial r_i(\alpha, z_i)}{\partial \alpha_i^2} dG_i(z_i) - \psi_i h_i \frac{\partial \ell_i(\alpha)}{\partial \alpha_i^2} G_i[\ell_i(\alpha)] + \psi_i(p_i + \varepsilon_i) \frac{\partial \ell_i(\alpha)}{\partial \alpha_i^2}[1 - G_i(\ell_i(\alpha))] = 0. \tag{16}$$

The proof of this theorem is straightforward and involves taking the first order necessary condition of the utility maximization problem of a product manager; hence, it is omitted. This theorem gives the necessary condition for a reward and an allocation rule to be implemented by revelation. It turns out that the combination of classes of functions $r_i(.)$ and $\ell_i(.)$ that satisfy equations (15) and (16) are quite narrow. One example of such functions are:

$$\begin{aligned} r_i(\alpha, z_i) = C_i(\alpha_{-i})[(\alpha_i^1 - \underline{z}_i^1) + (\bar{z}_i^1 - z_i^1) \ln \frac{\bar{z}_i^1 - \alpha_i^1}{\bar{z}_i^1 - \underline{z}_i^1} &+ \\ (\alpha_i^2 - \underline{z}_i^2) + (\bar{z}_i^2 - z_i^2) \ln \frac{\bar{z}_i^2 - \alpha_i^2}{\bar{z}_i^2 - \underline{z}_i^2}&], \text{ and} \end{aligned} \tag{17}$$

$\ell_i(\alpha)$ independent of α_i^1 and α_i^2 (for example, $\ell_i(\alpha) = K(\beta)/N$, where $K(\beta)$ is the total reported capacity and N is the number of product managers). Note that the bonus scheme in (17) is proper and is similar in form to that in (13) while the allocation rule is independent of the forecasts of the product manager i. We call this allocation rule to be the *constant allocation*. Note that under a proper bonus function, the constant allocation is the only allocation rule that can be implemented by revelation in dominant equilibrium under limited information. To see this, note that under a proper bonus function, the first term in equation (15) $\int_{\underline{z}_i}^{\bar{z}_i} \frac{\partial r_i(a, z_i)}{\partial \alpha_i^1} dG_i(z_i) = 0$. Therefore, the sum of other two terms in (15) must equal zero for all possible distribution function $G_i(.)$. The only way to do this is to set $\frac{\partial \ell_i(a)}{\partial \alpha_i^1} = 0$. Similar argument holds for equation (16) where we must have $\frac{\partial \ell_i(a)}{\partial \alpha_i^2} = 0$ under a proper bonus function. Thus, for dominant equilibrium under limited information, the allocation received by a product manager is independent of his forecast. Theorem 2 yields several additional important insights; we state them in the following corollaries.

Corollary 1: *In dominant equilibrium under limited information, a bonus payment is required to elicit the information truthfully from the managers. Profit sharing alone is not enough for eliciting truthful reporting.*

From the previous discussion we saw that only constant allocation can be implemented by revelation. Under a constant allocation rule, the allocation received by a PM is independent of his forecast. Under such an allocation rule the only reason for a PM to report truthfully is the bonus. Under a proper bonus function giving untruthful information reduces the expected payoff of a PM. Thus, a bonus is required for truthful reporting. Profit sharing alone (which means selecting an allocation rule without a bonus) is not enough. We next look at the possibility of implementing an individually responsive (IR) allocation mechanism by revelation. The following corollary summarizes our result.

Corollary 2: *An individually responsive (IR) allocation mechanism cannot be implemented by revelation in dominant equilibrium when the distribution of demand is unknown.*

Under an IR allocation mechanism, if one is receiving a positive allocation but wants more, one gets more unless one has already claimed all of the capacity. Thus, for an IR allocation we must have $\frac{\partial \ell_i(a)}{\partial a_i^1} > 0$, which makes it impossible to implement. This corollary tells us that proportional allocation (which is IR) cannot be implemented in dominant equilibrium in the case of no prior information. This supports the finding by Lee, Padmanabhan and Whang (1997) that a proportional allocation always gives rise to bull whip effect. Corollary 2 also implies that if an IR allocation rule is in place, the central coordinator will not get truthful reporting from the PMs. So far we have not addressed the issue of optimality in capacity allocation.

We next look at the possibility of implementing an optimal allocation mechanism (which maximizes the total profit from all product lines less the total incentive payments) by revelation. The following corollary summarizes our result.

Corollary 3: *An optimal allocation cannot be implemented by revelation in dominant equilibrium when the distribution of demand is unknown.*

The proof of this corollary is similar to that of Corollary 3 of MH (single statistics reporting). Implementing an optimal allocation requires the knowledge of the distributions $G_i(.)$. Knowing the first two moments are not enough. This means that under the condition of limited information it is not possible to achieve the optimal or the first best allocation.

Discussion

Table 1 compares the results of dominant equilibrium under limited information for single and multiple statistics reporting.. The most interesting aspect of the proper bonus function for multiple statistics reporting is its additive feature in the requested statistics. It also reduces to the proper bonus function of single statistics reporting when the second moment of the distribution is not requested. Apart from this, we do not find any significant difference between the single statistics reporting and multiple statistics reporting for the items of comparison in Table 1. We provide the following intuitive explanation. The central coordinator knows no information about the distributions in the case of limited information. In order to estimate a distribution of a random variable from its moments, one needs to know infinite number of moments. Our results in this section are valid for finite number of moments (note that Lemma 1 is valid for a finite vector T(.). Also note that a proper bonus function may not exist if the center requests an infinite number of moments). Thus, requesting an extra moment does not make any significant difference when the distribution is completely unknown.

4.2 Dominant Equilibrium Under Partial Information

In this section, we assume that the classes of distributions for demands and capacities are known to the center, but not the exact distribution. We model this by using a known distribution with an unknown parameter(s). Thus, the assumption of finite and known support of the random variables in Section 4.1 is no longer relevant in this section. Our analyses in this section will primarily focus on the Normal distribution with unknown parameters (both mean and variance are unknown). Let $Z_i \sim N(n_i, s_i^2), \forall i$ and $B_k \sim N(m_k, \overline{s}_k^2), \forall k$. However, these parameters are unknown to the center. The difference between the partial information cases of single statistics and multiple statistics reporting is the following. Under multiple statistics reporting, the distributions are normal with unknown parameters (both mean and variance are unknown) while under singles statistics reporting, the distributions are assumed to be Normal with *known* variance but *unknown* mean.

Theorem 3: *When the capacity is Normally distributed with unknown parameters, a quadratic bonus function of the following form is proper for MM k in dominant equilibrium for multiple statistics reporting:*

$$S_k(b, b_k) = c[2b_k^1 T_1(b_k) - (b_k^1)^2 + 2b_k^2 T_2(b_k) - (b_k^2)^2], \qquad (18)$$

where c is some constant independent of b_k.

This theorem tells us that the structure of the proper bonus function for the manufacturing managers is much simpler when the nature of the distribution is known (the case of partial information). It is interesting to compare the proper reward functions for single statistics reporting and multiple statistics reporting cases. For the single statistics case, the following are two examples of proper reward functions for the partial information case (as reported in MH):

$$S_k(b, b_k) = cb_k - 3c(b_k - b_k)^+ - c(b_k - b_k)^+ \text{, and} \tag{19}$$

$$S_k(b, b_k) = c[2b_k b_k - (b_k)^2] . \tag{20}$$

By comparing equation (18) and (20), we notice that the proper bonus function in multiple statistics case is additive in these reported moments. The functional form of (18) is also similar to that of (20). However, we do not obtain a proper reward function in the multiple statistics case that is similar in form to that of (19). We provide the following explanation. In the single statistics case, the center requested the expected value of the random variable B_k. Thus, the center could observe the outcome and penalize the individual managers such that a risk neutral manager will report his respective mean truthfully; this gave rise to the bonus function in (19). However, under multiple statistics reporting, the center requests the expected values of both B_k and B_k^2. Note that the random variable B_k^2 is never observed by the center, neither is it realized. Therefore, a bonus of the form (19) will not be appropriate for this case. The following theorem summarizes the structure of the bonus and allocation rules for truthful reporting by the PMs.

Theorem 4: *When the demands are Normally distributed with unknown parameters, a bonus function $r_i(.)$ and an allocation rule $\ell_i(.)$ for product manager i can be implemented by a revelation mechanism in dominant equilibrium if the $r_i(.)$ and $\ell_i(.)$ satisfy the following relationships:*

$$\int_{-\infty}^{\infty} \frac{\P r_i(a, z_i)}{\P a_i^1} e^{-(z_i - n_i)^2/2s^2} dz_i - \frac{\P \ell_i(a)}{\P a_i^1} y_i [(h_i + p_i + e_i) F[\frac{\ell_i(a) - n_i}{s_i}] - (p_i + e_i)] = 0 , \tag{21}$$

$$\int_{-\infty}^{\infty} \frac{\P r_i(a, z_i)}{\P a_i^2} e^{-(z_i - n_i)^2/2s^2} dz_i - \frac{\P \ell_i(a)}{\P a_i^2} y_i [(h_i + p_i + e_i) F[\frac{\ell_i(a) - n_i}{s_i}] - (p_i + e_i)] = 0 , \tag{22}$$

where F is the area under the standard Normal curve.

Theorem 4 specifies the first order necessary condition for a bonus function and an allocation to be implemented by revelation. Note that the parameters n_i and s_i are not known to the central coordinator. Thus, the responsibility of the central coordinator is to select a bonus function and an allocation rule such that the above first-order conditions are satisfied at $a_i^1 = n_i, a_i^2 = s_i^2 + n_i^2$. In other words, $a_i^1 = n_i, a_i^2 = s_i^2 + n_i^2$ are the solution to the system of equations (21)-(22). An example of such a bonus function and allocation rule is given by (23) and the modified lexicographic allocation of the form (24):

$$r_i(a, z_i) = c[2a_i^1 T_1(z_i) - (a_i^1)^2 + 2a_i^2 T_2(z_i) - (a_i^2)^2], \text{ and} \tag{23}$$

$$\ell_i(a) = min\{ K(b) - \sum_{j=1}^{i-1} \ell_j(a),\ a_i^1 + [a_i^2 - (a_i^1)^2]^{1/2} F^{-1}(\frac{p_i + e_i}{h_i + p_i + e_i}) \}. \tag{24}$$

Note that under a modified lexicographic rule, the PMs are ranked in some random manner independent of their forecast (say alphabetically) and are allocated production in accordance with this ranking. A PM with rank i receives the minimum of yet unallocated capacity and his "modified" forecast. This theorem tells us that by designing an appropriate bonus and an allocation rule, the center can extract information about the entire distribution when the form of the distribution is known. However, as explained by Lemma 1, the center can extract only finite number of moments. Thus, if the form of distribution is not known to the center, it will be able to approximate the distribution by requesting more and more number of moments, but will never be able to extract the entire distribution. Theorem 4, in addition, yields several important insights, which are stated in the following corollaries.

Corollary 4: *In dominant equilibrium under partial information for multiple statistics reporting, a bonus payment is essential to elicit the information truthfully from the manufacturing managers; however, it is not essential to elicit information truthfully from the product managers.*

This result is similar to the corresponding result of the single statistics case (Corollary 4 of MH). We showed in MH that it is possible for the central coordinator to design an allocation rule such that a PM will report information truthfully in absence of a bonus. Our current result establishes the same result to be true even when the variance of the Normal distribution is unknown. An example of such allocation rule is the modified lexicographic allocation rule described by equation (24). As shown in the proof of Corollary 4, under such an allocation rule the dominant strategy of each of the product managers is to report the expected values of requested statistics truthfully. Thus, in this situation, profit sharing alone is enough for truthful reporting. The difference between the modified lexicographic rules described in (24) and in MH (single statistics reporting) is that the allocation rule described in MH requires the knowledge of the standard deviation of the unknown distribution, while that described in (24) does not require the knowledge of the standard deviation. The intuitive explanation of why modified lexicographic allocation is truthful is the following. When PM i is ranked sufficiently high such that he will get the allocation of capacity, reporting $a_i^1 = n_i$, $a_i^2 = s_i^2 + n_i^2$ gives him the newsvendor optimal quantity. Thus, he has no incentive to over-report. Corollary 4 also implies that the modified lexicographic allocation rule is quite robust in the sense that it retains its revealing property even when the parameters of the Normal distribution are unknown. We next look at the possibility of implementing an individually responsive (IR) allocation rule by revelation.

Corollary 5: *Unlike the case of dominant equilibrium with limited information, an individually responsive (IR) allocation rule can be implemented by revelation in dominant equilibrium under partial information in multiple statistics reporting.*

The proof is by construction and is included in the Appendix. We have demonstrated the implementation of a linear allocation mechanism (which is IR) by appropriate choice of a bonus function. The presence of a bonus function, in addition to the profit function, in our model allows the implementation of an IR mechanism by revelation. The intuition behind this result is that the center can set the bonus for truthful forecasting to be sufficiently high so that it compensates a PM for the forgone profit in terms of product allocation. This result is similar to that of single statistics reporting case described in MH. However, there is an important distinction: a larger class of allocation rules (not just the IR allocation) will require a bonus for implementation by revelation for the multiple statistics reporting. We state this result formally in the following Corollary.

Corollary 5a: *In dominant equilibrium under partial information for multiple statistics reporting, a bonus payment is essential to elicit truthful information from a PM for any allocation rule that is solely a function of the outcome; i.e., the first moment of the unknown distribution.*

Note that IR allocation rules (for example, the linear and the proportional allocation rules) are special cases of the allocation rules defined by Corollary 5a. This corollary tells us that a much larger class of allocation rules will require a bonus for implementation by revelation under multiple statistics reporting. Note that the findings of Corollary 4 are supported by this corollary. In Corollary 4, we stated that the modified lexicographic allocation defined by (24) does not require a bonus for implementation. Note that this allocation rule is a function of the first two moments. This makes intuitive sense because, under partial information, more information about the unknown distributions is available to the central coordinator in the single statistics reporting as opposed to multiple statistics reporting (the variance is known in single statistics reporting). So far we have not addressed the issues of optimal allocation. Corollary 6 addresses this issue by studying the possibility of implementing an optimal allocation by revelation.

Corollary 6: *Given appropriate bonus functions for the PMs and MMs, an optimal allocation can be implemented by revelation in dominant equilibrium under partial information for multiple statistics reporting.*

We have defined the optimal allocation in Section 3.3 by equations (5)-(9). Given the appropriate bonus functions (as suggested by Corollary 6), $r_i(.)$ and $S_k(.)$ are no longer the decision variables in the optimal allocation problem defined by (5)-(9). Thus, the problem of the center is to find an allocation rule $\ell_i(a)$ that maximizes (4.5). Note that with a given $r_i(a, Z_i)$ and $S_k(b, B_k)$, the solution, ℓ_i^*, to the optimal allocation problem defined by Equations (5)-(9) is given by

$$F\left(\frac{\ell_i^* - n_i}{s_i}\right) = \frac{(1-y_i)(p_i + e_i) - l}{(1-y_i)(h_i + p_i + e_i)}, \tag{25}$$

where l is the shadow price of the capacity constraint and F is the area under standard Normal curve. Note that Equation (25) is also the optimal newsvendor solution under full information.

In our problem of partial information, consider the following allocation rule:

$$\ell_i(a) = a_i^1 + [a_i^2 - (a_i^1)^2]^{1/2} F^{-1}\left(\frac{(1-y_i)(p_i + e_i) - l}{(1-y_i)(h_i + p_i + e_i)}\right). \tag{26}$$

By comparing this allocation with the capacitated newsboy solution of (25), we can say that allocation rule (26) will implement an optimal allocation by revelation if $a_i^1 = n_i, a_i^2 = s_i^2 + n_i^2, \forall i$. Thus, to achieve this result, the center needs to design a bonus function $r_i(.)$ such that it, along with the allocation rule of (26), maximizes the expected payoff of PM i at $a_i^1 = n_i, a_i^2 = s_i^2 + n_i^2, \forall i$. As shown in the proof of Corollary 6, the following reward function achieves an optimal allocation:

$$r_i(a, z_i) = l \frac{y_i}{1-y_i}\left[(z_i - a_i^1) - \{a_i^2 - (a_i^1)^2\}^{1/2} F^{-1}\left(\frac{(1-y_i)(p_i + e_i) - l}{(1-y_i)(h_i + p_i + e_i)}\right)\right]. \tag{27}$$

Note that the allocation decisions obtained with (26) and (27) are newsvendor optimal. However, the resulting sum of profits is not the first best as it involves the coordination loss in terms of the reward payments. In fact, we cannot claim that the resulting profit is even second best. We have demonstrated the implementation of an optimal allocation for a *given* reward function. To claim that the resulting profit is second best we have to prove that the chosen reward functions result in minimum coordination loss among all possible reward functions. However, solving the optimization problem for arbitrary classes of functions $r_i(.)$, $S_k(.)$ and $\ell(.)$ is extremely difficult. Therefore, we had to restrict Corollary 6 for *given* reward functions. However, there is an important distinction between the implementation of optimal allocation in single statistics reporting and in multiple statistics reporting. Note that the bonus in multiple statistics reporting defined by equation (27) is *not* proper. The bonus for implementation of optimal allocation in single statistics reporting was proper. In fact, a more powerful result can be proved for the multiple statistics reporting case that we state in the following corollary.

Corollary 6a: *In dominant equilibrium under partial information for multiple statistics reporting, there exists no proper bonus function that allow the implementation of an optimal allocation by revelation.*

This corollary is quite powerful in the sense that it tells us that we lose the desirable properties of the bonus function if we want to implement an optimal allocation by revelation. Nonetheless, it is possible to implement an optimal allocation by revelation. This corollary also allows us to compare and contrast the implementation of optimal allocation under different conditions of information availability; Table 2 accomplishes this comparison. Note that we have considered three different conditions of information availability, which are summarized in the first column of this table. The example cases are provided in the second column. We can see that as more information is made available to the center, the implementation of optimal allocation is becomes easier. For example, the cases of limited information do not let the central coordinator implement an optimal allocation by revelation. When the center knows that the demand distributions to be Normal but does not know the parameters, an optimal allocation can be implemented with the help of a bonus that is not proper. When the variance of the normal distributions are known, the center can restrict itself to a special class of bonus functions, namely, the proper bonus functions.

From Table 2, we see that the optimal allocation cannot be implemented by revelation under the condition of limited information (for both single and multiple statistics reporting). We have also shown that the modified lexicographic allocation is truthful under partial information. However, we could not make any claims about the residual profit of the firm under the lexicographic allocation rule. Note that under a lexicographic allocation, the product managers are ranked in a random manner independent of their order size. Thus, it is possible that a product with a very high margin will not be ranked sufficiently high on the priority list to be allocated capacity. Under such situation, the residual profit of the firm might be greater under a different allocation rule, even though the allocation rule might not be truth revealing. This naturally makes us wonder whether an allocation being truthful is always desirable (or if an allocation rule being manipulable is always undesirable). Our model, in its current form, does not address these issues. Our current effort is to identify the allocation rules that are truthful and the ones that are manipulable. For the semiconductor manufacturer, this is useful not only for making the allocation decisions, but also for making outsourcing decisions.

We address the issue of maximizing the residual return for the firm while discussing the possibility of implementing an optimal allocation by revelation. Note that we have used the *revelation principle* of economics to restrict our search. Our result of implementing an optimal allocation by revelation is restricted for a *given* bonus function. The optimal allocation problem defined by (5)-(9) is extremely difficult to solve for arbitrary class of bonus functions $r_i(.)$ and $S_k(.)$. Therefore, we had to restrict our search to a *given* bonus function. In fact, it is a difficult problem when the system control is centralized, as has become evident from extensive studies in a variety of contexts. Some examples include single-warehouse/multi-retailer settings (e.g., Eppen 1979; Eppen and Schrage, 1981; inventory systems with different customer classes (e.g., Nahmias and Demmy, 1981); and queuing systems with multiple servers or customer classes (e.g., Gilbert and Weng, 1997).

Discussion

Table 3 compares the results from single statistics reporting and multiple statistics reporting. The most interesting aspect of the proper bonus function for multiple statistics reporting is its additive feature in the requested statistics. It also reduces to the proper bonus function of single statistics reporting when the second moment of the distribution is not requested. However, unlike the case of single statistics reporting, we did not obtain a piece-wise linear bonus function in multiple statistics reporting. Other major differences between single and multiple statistics reporting under partial information have been summarized by Corollaries 5a and 6a.

Another interesting conclusion can be drawn from our analyses. Modified lexicographic allocation is quite robust in the sense that it can be implemented by revelation even when the parameters of the Normal distribution are unknown. It is clear from the preceding discussion that dominant equilibrium with partial information allows us significant flexibility over the case of dominant equilibrium under limited information regarding the implementation of different allocation mechanisms.

4.3 Bayesian Nash Equilibrium

The Bayesian equilibrium does not allow us any additional flexibility in terms of implementing an IR allocation or an optimal allocation. In addition, a Bayesian equilibrium is less robust than a dominant equilibrium. Table 4 summarizes our results for Bayesian equilibrium. For the reasons of brevity, all analyses are presented in Appendix B.

5.0 GENERALIZATION TO *n*-STATISTICS REPORTING

In this section, we show that our results for two statistics reporting are generalizable to any arbitrary but finite number of statistics. This is a significant result as this allows the central coordinator to get arbitrarily close to the unknown distributions by requesting more and more information. Our generalization is described under the following subsections.

5.1 Model Definition and Notation

We modify the notation of Section 3.1 for n-statistics reporting. The center requests the expected values of n transformations, $T(.) = (T_1(.), T_2(.), \ldots, T_n(.))$, of the random variables $Z_i(.)$, $i = 1,2, .., N$, and $B_k(.)$, $k = 1,2, \ldots, M$.

a_i^j = report of j^{th} transformation from PM i, $j = 1,\ldots,n$,

b_k^j = report of j^{th} transformation from MM k, $j = 1,\ldots, n$,

$b_k = (b_k^1,\ldots,b_k^n)$, $b = (b_1,\ldots,b_M)$, $b_{-k} = (b_1,\ldots,b_{k-1},b_{k+1},\ldots,b_M)$,

$a_i = (a_i^1, a_i^2,\ldots,a_i^n)$, $a = (a_1,a_2,\ldots,a_N)$, $a_{-i} = (a_1,a_2,\ldots,a_{i-1},a_{i+1},\ldots,a_N)$.

The utility maximization problems of MM k, k = 1, 2, …, M, and PM i, i = 1, 2, …, N, under the assumption of risk neutrality, is given respectively by

$$\max_{b_k} \; E_{F_k} S_k(b, B_k), \text{ and} \tag{1'}$$

$$\max_{a_i} \; E_{G_i} R_i(a, Z_i). \tag{4'}$$

Note that the utility maximization problems must satisfy the first order necessary condition for each components of the vectors $a_i = (a_i^1, a_i^2, \ldots, a_i^n)$ and $b_k = (b_k^1, \ldots, b_k^n)$. The problem faced by the center is to choose the allocation mechanism $\ell_i(\alpha)$ and the incentives $S_k(\beta, b_k)$ and $r_i(\alpha, z_i)$ such that truth telling is the equilibrium strategy for all managers.

Note that the definitions responsive and accurate bonus functions remain the same as that of Section 3.4. The definition of rational and revealing is extended for each of the reported statistics. This means that the reward function $S_k(., .)$ is revealing when it satisfies the following conditions simultaneously:

$$\frac{\partial}{\partial b_k^j}\{E\,S_k(b, B_k)\} = 0, \ \forall j, \forall k. \tag{10′}$$

Similar definition holds for the bonus of the product managers, $r_i(a, Z_i)$.

5.2 Analyses

Our analyses of the limited information case (Section 4.1) were based on Lemma 1. Lemma 1′ provides a generalization of that lemma.

Lemma 1 ¢ *Let $X \in [a,b]$ be a random variable with distribution function F(.). Let $T_1(x)$, $T_2(x)$, …, $T_n(x)$, for finite n, be continuous and strictly monotonic functions of x and $T(x) = (T_1(x), T_2(x), \ldots, T_n(x))$. Also let $t = (t_1, t_2, \ldots, t_n)$ denote another vector of finite dimension. A continuously differentiable function $g(x)$: $[a, b] \rightarrow R$, satisfies the condition* $\int_a^b g(x)\,dF(x) = 0$, $\forall F$ *such that $\int_a^b T(x)\,dF(x) = t$, if and only if there exists a vector $k = (k_1, k_2, \ldots, k_n)$, independent of x, such that $g(x) = \sum_{i=1}^{n} k_i\,[T_i(x) - t_i]$.*

The proof is included in the Appendix. It is clear from this lemma that the additive form of bonus functions in requested statistics will be valid for any finite generalization of the two statistics case. This result leads to the following structure of the proper bonus function for MM k.

$$S_k(b, b_k) = u(b_k) - \nabla u(b_k) \cdot [b_k - T(b_k)] + \overline{C}(b_k), \tag{11'}$$

for some appropriate function u(.) and constant $\overline{C}(b_k)$, where $\nabla u(.)$ represent the gradient of $u(.)$ and the term $\nabla u(b_k) \cdot [T(b_k) - b_k]$ represents scalar product of two vectors. Clearly, this is the generalization of Theorem 1 of Section 5.2.1. The proof is similar to that of Theorem 1 except that it involves simultaneous solution of *n* first order conditions instead of two equations. An example of the function *u(.)* that gives rise to a strongly quasiconcave reward function is given by:

$$u(b_k) = \sum_{j=1}^{n} (\overline{B_k^j} - b_k^j)[ln(\overline{B_k^j} - b_k^j) - 1]. \tag{12'}$$

Substituting (12′) into (11′) and simplifying, leads to the following *proper* reward function for MM k:

$$S_k(b, b_k) = C_k(b_{-k}) \left[\sum_{j=1}^{n} (b_k^j - \underline{B_k^j}) + (\overline{B_k^j} - T_j(b_k)) ln \frac{\overline{B_k^j} - b_k^j}{\overline{B_k^j} - \underline{B_k^j}} \right]. \tag{13'}$$

From the discussions so far it is clear that the results of Section 4.1 can be generalized to any finite number of statistics. The analyses for the product managers are similar and yield identical insights. Our analyses for the case of partial information assume the unknown distributions to be Normal. Therefore the question of generalization beyond two statistics does not arise.

6.0 SUMMARY AND DIRECTIONS FOR FUTURE RESEARCH

In this paper, we studied contracting over multiple parameters in context of forecasting and capacity allocation. This is a generalization of our previous work, MH, on single statistics reporting. The majority of the operations management literature on contracting and supply chain coordination assume only one unknown parameter over which contract is made (for example, Cachon and Lariviere, 1996, 1997; Corbett, 1996, 1997). We have shown that it is possible to design a mechanism that elicits information truthfully about multiple parameters of a distribution simultaneously. Our results for two statistics reporting are generalizable to any arbitrary finite number of statistics. The most interesting feature of the proper reward function in multiple statistics reporting is its additive form in the requested statistics, with a similar functional form in each statistic. This is valid for both limited and partial information cases. It is shown that under the condition of limited information (when nothing is known about the distributions and no prior is available), requesting an extra moment is of limited use. Under the condition of partial information, we did not obtain a piece-wise linear bonus function in multiple statistics reporting. The major differences between single and multiple statistics reporting under partial information have been summarized by Corollaries 5a and 6a. Another interesting conclusion can be drawn from our analyses. Modified lexicographic allocation is quite robust in the sense that it can be implemented by revelation even when the parameters of the Normal distribution are unknown. It is also shown that dominant equilibrium with partial information allows us significant flexibility over the case of dominant equilibrium under limited information regarding the implementation of different allocation mechanisms. However, Bayesian implementation offers virtually no advantage over a dominant strategy implementation. The work was motivated by the experiences of a US-based semiconductor manufacturer. However, the framework we have proposed is quite general and can be applied to any allocation problem.

We assumed risk neutrality to kep the results analytically tractable. We note that virtually all coordination models reported in operations management literature (Porteus & Whang, 1991; Cachon & Lariviere, 1996, Corbett, 1996) assume risk neutrality. The main contribution of our model is in considering a contracting problem over multiple parameters. Despite our specific motivation from semiconductor manufacturing, the analytical results presented in this paper are general enough to be used in other coordination problems. We identify the following directions for our future research.

- *Risk Averse Managers*: As mentioned above, the assumption of risk neutrality is rather strong one. A manufacturing manager tends to deflate his forecast to cover for the uncertainties. The best way to model this situation is to consider risk averse managers.
- *Exploring Market Mechanisms:* We have mentioned before that our problem contains a market-like structure. We have also shown in Section 4.3 that the presence of multiple MMs lowers the coordination costs in an expected sense. A natural extension to this idea is to look for a market mechanism based on auctions/bidding to allocate the available capacities. The specific issues to look at will be the following. What are the structure of the auction (for example a PM bids for capacities or an MM submits asks for available capacities)? What are the allocation decisions and coordination costs under an auction-based method? How does an auction-based method compare with optimization-based method (our current approach)?
- *Extension to Random Yield Problems*: We have stated in Section 1 that the capacities are random in our problem, as the yields are random in semiconductor manufacturing. This, naturally, give rise to the idea of studying the random yield problem under asymmetric information. To our knowledge, this has not been looked at in the literature.

REFERENCES

Atkinson, A.A., "Incentives, Uncertainty and Risk in the Newsboy Problem," *Decision Sciences*, Vol. 10, pp. 341-357, 1979.

Bazaraa, M.S., C.M. Shetty, *Nonlinear Programming: Theory and Algorithms*, John Wiley, NY, 1979.

Cachon, G.P., M.A. Lariviere, "Capacity Choice and Allocation: Strategic Behavior and Supply Chain Performance," Working Paper, Duke University, 1996.

Cachon, G.P., M.A. Lariviere, "Capacity Allocation Using Past Sales: When to Turn-and-Earn," Working Paper, Duke University, 1997.

Clark, A.J., H. Scarf, "Optimal Policies for a Multi-Echelon Inventory Problem," *Management Science*, Vol. 6, No. 4, pp. 475-490, 1960.

Corbett, C., "Consignment Stock and Demand Variability in an Imperfect Supply Chain Partnership," Unpublished dissertation, INSEAD, 1996.

Eppen, G.D., "Effects of Centralization on Expected Costs in a Multi-Location Newsboy Problem," *Management Science*, 25, 5, pp. 498-501, 1979

Eppen, G.D., L. Schrage, "Centralized Ordering Policies in a Multi-Warehouse System with Leadtimes and Random Demand," In L.B. Schwarz, (Ed.), *Multi-Level Production/Inventory Control System: Theory and Practice*, pp. 51-67, North-Holland, 1981.

Fudenberg, D., J. Tirole, *Game Theory*, MIT Press, Cambridge, MA, 1991.

Gilbert, S.M., Z.K. Weng, "Incentive Effects Favor Non-Consolidating Queues in a Service System: The Principal-Agent Perspective, Working Paper, Case Western Reserve University, 1997.

Gonik, J., "Tie Salesmen's Bonuses to Their Forecasts," *Harvard Business Review,* 56, pp. 116-123, May-June, 1978

Grout, J.R., D.P. Christy, "An Inventory Model of Incentives for On-Time Delivery in Just-in-Time Purchasing Contracts," *Naval Research Logistics,* Vol. 40, pp. 863-877, 1993.

Grout, J.R., "A Model of Incentive Contracts for Just-In-Time Delivery," Working Paper, Southern Methodist University, 1996.

Groves, T., M. Loeb, “Incentives in a Divisionalized Firm,” *Management Science*, Vol 25, No. 3, pp. 221-230, 1979.

Harris, M., C.H. Kriebel, A. Raviv, “Asymmetric Information, Incentives and Intrafirm Resource Allocation,” *Management Science* Vol. 28, No. 6, pp. 604-620, 1982.

Hauser, J.R., D.I. Simester, B. Wernerfelt, “Internal Customers and Suppliers,” Working Paper, MIT, 1994.

Kandel, E., “The Right to Return,” *Journal of Law and Economics*, April, 1996.

Lee, H., S. Whang, “Decentralized Multi-Echelon Inventory Control Systems: Incentives and Information,” Working Paper, Stanford University, 1994.

Lee, H., P. Padmanabhan, S. Whang, “Information Distortion in a Supply Chain: The Bullwhip Effect,” *Management Science*, Vol. 43, No. 4, pp.546-558, 1997.

Mallik, S. and P.T. Harker, "Coordinating Supply Chains With Competition: Capacity Allocation in Semiconductor Manufacturing," Working Paper, Fishman-Davidson Center, The Wharton School, University of Pennsylvania, 1998.

Nahmias, S., W.S. Demmy, “Operating Characteristics of an Inventory System with Rationing,” *Management Science*, 27, pp. 1236-1244, 1981.

Milgrom, P.R., R.J. Weber, “Distributional Strategies for Games with Incomplete Information,” *Mathematics of Operations Research*, Vol. 10, pp. 619-631, 1986.

Osband,K., S. Reichelstein, “Information-Eliciting Compensation Schemes,” *Journal of Public Economics*, 27, pp. 107-115, 1985.

Porteus, E.L., S. Whang, “On Manufacturing/Marketing Incentives,” *Management Science*, Vol. 37, No. 9, pp. 1166-1181, 1991.

Tsay, A.A., S. Nahmias, N. Agrawal, “Modeling Supply Chain Contracts: A Review,” In S. Tayur et al., (Eds.), *Quantitative Models for Supply Chain Management*, pp. 299-336, Kluwer Academic Publishers, 1999.

Van Ackere, A., “The Principal/Agent Paradigm: Its Relevance to Various Functional Fields,” *European Journal of Operational Research*, Vol. 70, pp. 83-103, 1993.

Whang, S., “Coordination in Operations: A Taxonomy,” *Journal of Operations Management*, 12, pp. 413-422, 1995.

Table 1: Comparison of Dominant Equilibrium Results (under limited information) for the Single Statistics and Multiple Statistics Reporting

Item of Comparison	*Single Statistics Reporting*	*Multiple Statistics Reporting*
Structure of Proper Bonus	Linear in outcome, Log in forecast	Additive in the requested statistics with linear in outcome, log in forecast for each statistics
Bonus for MM	Required for truthful reporting	Required for truthful reporting
Bonus for PM	Required for truthful reporting	Required for truthful reporting
IR Allocation	Manipulable	Manipulable
Optimal Allocation	Manipulable	Manipulable
Truthful Allocation	Constant Allocation	Constant Allocation

Table 2: Comparison of Optimal Allocation Results under Different Conditions of Reporting and Information Availability.

Information Availability	*Example Case(s)*	*Optimal Allocation Implementation*
Nothing is known about the demand distributions	Limited Information • Single statistics reporting • Multiple statistics reporting	Manipulable
Demands Normally distributed with unknown parameters	Partial Information • Multiple statistics reporting	• Truthful with bonus • Bonus is *not* proper
Demands Normally distributed with unknown mean, known variance	Partial Information • Single statistics reporting	• Truthful with bonus • Bonus *is* proper

Table 3: Comparison of Dominant Equilibrium Results (under partial information) for the Single Statistics and Multiple Statistics Reporting

Item of Comparison	*Single Statistics Reporting*	*Multiple Statistics Reporting*
Structure of Proper Bonus	• Piece-wise linear • Quadratic in report	Additive in the requested statistics with quadratic in report
Bonus for MM	Required for truthful reporting	Required for truthful reporting
Bonus for PM	Not required for truthful reporting	Not required for truthful reporting
IR Allocation	Truthful with bonus	• Truthful with bonus • A larger class requires bonus
Optimal Allocation	• Truthful with bonus • Bonus is proper	• Truthful with bonus • Bonus is not proper
Truthful Allocation	Modified lexicographic allocation	• Modified lexicographic allocation • Knowledge of s_i not required

Table 4: Summary of Bayesian Equilibrium Results under Multiple Statistics Reporting

Item of Comparison	*Bayesian Equilibrium Results*	*Dominant Equilibrium Results*
Bonus for MM	Required	Required
Bonus for PM	Not Required	Not Required
IR Allocation	Implementable with bonus	Implementable with bonus
Optimal Allocation	Implementable with bonus	Implementable with bonus

TECHNICAL APPENDIX

Title: Contracting Over Multiple Parameters: Capacity Allocation in Semiconductor Manufacturing

Authors: Suman Mallik and Patrick T. Harker

Index:

A. Definition of equilibrium concepts
B. Analyses for Bayesian equilibrium
C. Proofs for all theorems, lemmas and corollaries

APPENDIX A: DEFINITION OF EQUILIBRIUM CONCEPTS

We consider two different solution concepts, the dominant strategy equilibrium and Bayesian Nash equilibrium. We formally define these two concepts here.

Dominant Strategy Equilibrium

We consider the MMs first. Recall that $b_k = (b_k^1, b_k^2)$. Given an incentive scheme $S_k(.)$ for MM k, $\beta_k \in \Delta_k$ is the report of MM k; $\beta_{-k} \in \prod_{j\neq k} \Delta_j$ are the reports of all other MMs and b_k is the realization of the random capacity B_k. A vector of reported forecasts $x^* = \{x_1^*, x_2^*, \ldots, x_M^*\}$ form a dominant equilibrium when for k = 1, 2, ..., M :

$$E_{F_k} S_k(x_k^*, \beta_{-k}; B_k) \geq E_{F_k} S_k(\beta_k, \beta_{-k}; B_k), \forall \beta_k \in \Delta_k, \forall \beta_{-k} \in \prod_{j\neq k} \Delta_j \text{ .} \quad \text{(A1)}$$

Now consider the product managers. A vector of reported forecasts $x^* = \{x_1^*, x_2^*, \ldots, x_N^*\}$ form a dominant equilibrium when for i = 1, 2, ..., N :

$$E_{G_i} R_i(x_i^*, \alpha_{-i}; Z_i) \geq E_{G_i} R_i(\alpha_i, \alpha_{-i}; Z_i), \forall \alpha_i \in \Lambda_i, \forall \alpha_{-i} \in \prod_{j\neq i} \Lambda_j \text{ .} \quad \text{(A1}')$$

The sets Δ_k and Λ_i are defined by the support of the random variables corresponding to the demands and capacities.

Bayesian Nash Equilibrium

We begin by considering the MMs. In our formulation, the *type* of each player (MM) is distinguished by the respective capacity *distributions*. We need the following additional notations for a formal definition of Bayesian equilibrium. Assume that b_k^1 belongs to a closed interval $Q_k^1, \forall k$; and b_k^2 belongs to a closed interval $Q_k^2, \forall k$. The vector of realized types $b = (b_1, \ldots, b_M)$ is an element of $Q = \prod_{j=1}^{M} Q_j^1 \times Q_j^2$. Let $\gamma(\beta)$ be the joint distribution of all reports of all MMs and $\gamma_k(\beta_{-k})$ be the joint distribution of reports of all managers but k, conditional on k's report β_k. In line with the existing literature (Fudenberg and Tirole, 1991), we assume $\gamma(\beta)$, and hence $\gamma_k(\beta_{-k})$, to be common knowledge. Given the reward function $S_k(.)$, let $\bar{S}_k(x_k,\beta_{-k};b_k)$ be the expected reward of MM k when his reported forecast is x_k, the realization of random capacity is b_k, and the reported forecasts of all other managers is β_{-k}, i.e.

$$\bar{S}_k(x_k,b_{-k};b_k)=\int_{Q_{-k}} S_k(x_k,b_{-k};b_k)\,d\,g_k(b_{-k}).$$

A vector of reported forecasts $x^*=\{x_1^*,x_2^*,\ldots,x_M^*\}$ form a Bayesian Nash equilibrium, or simply Bayesian equilibrium, when for k = 1, 2, ..., M:

$$E_{F_k}\bar{S}_k(x_k^*,x_{-k}^*;B_k)\geq E_{F_k}\bar{S}_k(b_k,x_{-k}^*;B_k),\forall b_k\in Q_k^1\times Q_k^2,\ \forall x_{-k}\in\prod_{j\neq k}Q_j^1\times Q_j^2\,. \qquad \text{(A2)}$$

Similarly, assume that a_i^1 belongs to a closed interval $W_i^1,\forall i$; and a_i^2 belongs to a closed interval $W_i^2,\forall i$. The vector of realized types $a=(a_1,\ldots,a_N)$ is an element of $W=\prod_{j=1}^{N}W_j^1\times W_j^2$.

Let $\kappa(\alpha)$ be the joint distribution of all reports of all PMs and $\kappa_i(\alpha_{-i})$ be the joint distribution of reports of all PMs but i, conditional on i's report. A vector of reported forecasts $x^*=\{x_1^*,x_2^*,\ldots,x_N^*\}$ form a Bayesian Nash equilibrium, or simply Bayesian equilibrium, when for i = 1, 2, ..., N:

$$E_{G_i}\bar{R}_i(x_i^*,x_{-i}^*;Z_i)\geq E_{G_i}\bar{R}_i(a_i,x_{-i}^*;Z_i),\forall a_i\in W_i^1\times W_i^2,\ \forall x_{-i}\in\prod_{j\neq i}W_i^1\times W_i^2\,, \qquad \text{(A2')}$$

where, $\bar{R}_i(a_i,a_{-i};z_i)=\int_{W_{-i}} R_i(a_i,a_{-i};z_i)\,d\,k_i(a_{-i})$.

APPENDIX B: BAYESIAN EQUILIBRIUM ANALYSES

We have defined the Bayesian Nash equilibrium in Appendix A. In this section, as in Section 4.2, we assume that the center knows the classes of distributions for demands and capacities but not the exact distributions. Our discussion in this Section focuses on the Normal distribution with unknown parameters. All managers and the center know the common priors $g(.)$ and $k(.)$ for the joint distributions of the requested moments. Thus, the difference between the partial information analyses (Section 4.2) and the Bayesian analyses are the following. In Bayesian analyses the center, in addition to knowing that the distributions are Normal, knows the common priors $g(.)$ and $k(.)$. We note that any dominant equilibrium solution is also a Bayesian solution. Bayesian equilibrium is a much less demanding (and hence, less robust) solution concept than the dominant equilibrium. The rational behind studying Bayesian equilibrium solution is to see whether we can get any additional and interesting solution (in terms of not providing a bonus or implementing additional classes of allocation rules) to our hidden information problem. The following theorem provides structural result on Bayesian equilibrium.

Theorem B1: *Under an accurate reward function, there exists*

(a) a pure strategy Bayesian Nash equilibrium in the forecasting games when the bonus functions are continuous and quasiconcave in b_k^2 *and* a_i^2 *respectively; and*

(b) a purification of each strategy of each player.

This is a generalization of a similar theorem (Theorem B1) described in MH. Note that we need additional restrictions (compared to single statistics reporting) on the bonus functions in order for the equilibrium to exist. This is specified in terms of the quasiconcavity of the reward functions in b_k^2 and a_i^2. The assumption of closedness of the sets Q and W are also required to guarantee the existence of the equilibrium. We have defined the terminology *purification* in MH. Part (b) of the theorem tells us that by restricting our attention to the pure strategies, we do not lose any generality or insights of the problems. However, the restrictive assumption of quasiconcavity was required to prove the purification result. We next look at the results concerning the bonus functions and allocation rules. The following is true for a manufacturing manager.

- *A bonus is required for a MM to elicit truthful information under Bayesian equilibrium.*

This is true as the payoff of a manufacturing manager consists only of the bonus. Thus, the central coordinator needs to provide the bonus in order to extract the private information. We next look at the product managers. The first order necessary condition for the utility maximization problem of PM i is given by:

$$\int_{\Omega_{-i}} \{ \int_{-\infty}^{\infty} \frac{\partial \rho_i(\alpha, z_i)}{\partial \alpha_i^1} e^{-(z_i-\nu_i)^2/2\sigma^2} dz_i - \frac{\partial \ell_i(\alpha)}{\partial \alpha_i^1} \psi_i [(\eta_i+\pi_i+\varepsilon_i)\Phi[\frac{\ell_i(\alpha)-\nu_i}{\sigma_i}] - (\pi_i+\varepsilon_i)]\} d\kappa_i(\alpha_{-i}) = 0, \qquad \text{(B1)}$$

$$\int_{\Omega_{-i}} \{ \int_{-\infty}^{\infty} \frac{\partial \rho_i(\alpha, z_i)}{\partial \alpha_i^2} e^{-(z_i-\nu_i)^2/2\sigma^2} dz_i - \frac{\partial \ell_i(\alpha)}{\partial \alpha_i^2} \psi_i [(\eta_i+\pi_i+\varepsilon_i)\Phi[\frac{\ell_i(\alpha)-\nu_i}{\sigma_i}] - (\pi_i+\varepsilon_i)]\} d\kappa_i(\alpha_{-i}) = 0. \qquad \text{(B2)}$$

Noting that any dominant equilibrium solution is also a Bayesian solution, we readily obtain the following result.

- *A bonus is required for a PM to elicit truthful information under Bayesian equilibrium.*

We have shown in Section 4.2 that modified lexicographic allocation, defined by (24) can be implemented by revelation without a bonus. The same solution carries through in the Bayesian solution. It is easy to see that (24) satisfies (B1)-(B2). The following corollaries summarize our results for IR allocation and optimal allocation under Bayesian equilibrium.

Corollary B2: An *individually responsive allocation can be implemented by revelation in Bayesian equilibrium under multiple statistics reporting. A bonus is, however, required for its implementation.*

Corollary B3: Given *an appropriate bonus function, an optimal allocation can be implemented by revelation in Bayesian equilibrium under multiple statistics reporting. A bonus is, however, required for its implementation.*

The proofs of the above corollaries are included in Appendix C. The intuition behind the above results is similar to those of Section 4.2. Under an IR allocation rule with capacity binding, there is at least one PM who is receiving less than the desired amount. Therefore, he has an incentive to inflate his order. However, doing so reduces his expected payoff in presence of an accurate bonus. Thus, a bonus will be required to implement an IR allocation by revelation. The results of Bayesian analyses have been summarized in Table 4. Comparing these results with the dominant equilibrium results (under partial information), we see that Bayesian equilibrium offers no additional flexibility in terms of not providing a bonus for truthful reporting or implementing additional classes of allocation rules by revelation. Therefore, the central coordinator is better of implementing a dominant equilibrium. We obtained a similar result in case of Bayesian solution for the single statistics reporting.

APPENDIX C: PROOFS OF THEOREMS

Proof of Lemma 1 and 1 ¢

Lemma 1 and 1′ are the same except that Lemma 1′ is valid for an arbitrary but finite *n*, while lemma 1 is valid for n = 2. Therefore, we provide the proof for an arbitrary *n*. Substituting n = 2 yield the proof for Lemma 1. The "if" part in both the lemmas are trivially true. Consider the "only if" part. Consider the i^th component of the condition $\int_a^b T(x)dF(x) = t$, for i = 1,2,…,n. Here F(.) satisfies $\int_a^b T_i(x)dF(x) = t_i$. Choose any two points (x_0, x_1) ∈ [a, b] such that $T_i(x_0) < t_i < T_i(x_1)$. Note that under the assumption that $T_i(x)$ is continuous and strictly monotonic in x and that t_i is the expected value of $T_i(x)$, it is clearly possible to choose (x_0, x_1) ∈ [a, b] such that $T_i(x_0) < t_i < T_i(x_1)$. Assume that $\frac{g(x_1)}{g(x_0)} \neq \frac{T_i(x_1)-t_i}{T_i(x_0)-t_i}$. Since X is *any* random variable, we construct the following r.v.

$$X = \begin{cases} x_1 \;\; w.p. \; \dfrac{t_i - T_i(x_0)}{T_i(x_1)-T_i(x_0)} \\ \\ x_0 \;\; w.p. \; \dfrac{T_i(x_1)-t_i}{T_i(x_1)-T_i(x_0)} \end{cases} \qquad \text{(C1)}$$

Note that with this construction, the condition $E_F\,[T_i(x)] = t_i$ is satisfied. Now,

$E_F\, g(X) = g(x_1)\frac{t_i - T_i(x_0)}{T_i(x_1)-T_i(x_0)} + g(x_0)\frac{T_i(x_1)-t_i}{T_i(x_1)-T_i(x_0)} \neq 0$, as $\frac{g(x_1)}{g(x_0)} \neq \frac{T_i(x_1)-t_i}{T_i(x_0)-t_i}$. Hence we have a contradiction. Therefore we must have $\frac{g(x_1)}{g(x_0)} = \frac{T_i(x_1)-t_i}{T_i(x_0)-t_i}$; implying $g(x) = k_i\,[T_i(x)-t_i]$, i = 1,2,…,n, where, k_i is a constant independent of x. Suppose that it is possible to find $\overline{F}(.)$ and $\overline{g}(.)$ such that $E_{\overline{F}}[\overline{g}(x)] = 0$, but $\overline{g}(x) \neq k_i\,[T_i(x)-t_i]$. For this $\overline{g}(x)$, choose F(x) to be the cdf defined by (C1). Clearly, $E_F[\overline{g}(x)] \neq 0$. Note that our lemma requires $E_F[g(x)] = 0, \forall F$. Therefore the construction of $\overline{g}(x)$ is incorrect. We have shown that $\int_a^b g(x)\,dF(x) = 0$, $\forall F$ such that $\int_a^b T_i(x)dF(x) = t_i$, if and only if $g(x) = k_i\,[T_i(x) - t_i]$, $\forall i$. Now consider that F(.) satisfy all n conditions simultaneously, i.e., $\int_a^b T(x)dF(x) = t$. This means that F(.) satisfies the condition

$$\int_a^b [\sum_{i=1}^{n} m_i T_i(x)]\, dF(X) = \sum_{i=1}^{n} m_i t_i\,, \tag{C2}$$

for constants m_1, …m_n. Repeating the argument presented above we can say that $\int_a^b g(x)\,dF(x) = 0$, $\forall F$ such that (C2) holds if and only if:

$g(x) = M\,[\sum_{i=1}^{n} m_i T_i(x) - \sum_{i=1}^{n} m_i t_i\,]$, where, M is a constant independent of x. Rearranging and writing $k_i = m_i M$, we get, $g(x) = \sum_{i=1}^{n} k_i\,[T_i(x) - t_i\,]$. **Q.E.D.**

<u>Proof of Theorem 1</u>

If: Follows trivially

Only if: The problem faced by the MM k is given by: $\underset{b_k^1, b_k^2}{Max} E_{F_k} S_k(b, B_k)$. The objective of the center is to choose the bonus function $S_k(.)$ such that it is proper. The bonus is revealing means it must satisfy the conditions

$$\frac{\partial}{\partial b_k^1}\{E\,S_k(b, B_k)\} = 0, \forall k, \text{ and} \tag{C3}$$

$$\frac{\partial}{\partial b_k^2}\{E\,S_k(b, B_k)\} = 0, \forall k\,. \tag{C4}$$

(C3) implies that $\frac{\partial}{\partial b_k^1}\int S_k(b, b_k)dF(b_k) = 0, \forall F$. Using Lemma 1 we get,

$$\frac{\partial S_k(b, b_k)}{\partial b_k^1} = k_1(b)[T_1(b_k) - b_k^1] + k_2(b)[T_2(b_k) - b_k^2]\,. \tag{C5}$$

Integrating (integrate the first term by parts) we get

$$S_k(b,b_k) = [T_1(b_k) - b_k^1]\int k_1(b)\,db_k^1 \quad + \quad [T_2(b_k) - b_k^2]\int k_2(b)\,db_k^1 \\ + \int[\int k_1(b)\,db_k^1]\,db_k^1 \quad + \quad c(b_k). \tag{C6}$$

Similarly, (C4) implies that $\frac{\partial}{\partial b_k^2}\int S_k(b,b_k)\,dF(b_k) = 0, \forall F$. Using Lemma 1 we get,

$$\frac{\partial S_k(b,b_k)}{\partial b_k^2} = \bar{k}_1(b)[T_1(b_k) - b_k^1] + \bar{k}_2(b)[T_2(b_k) - b_k^2]. \tag{C7}$$

Integrating,

$$S_k(b,b_k) = [T_1(b_k) - b_k^1]\int \bar{k}_1(b)\,db_k^2 \quad + \quad [T_2(b_k) - b_k^2]\int \bar{k}_2(b)\,db_k^2 \\ + \int[\int \bar{k}_2(b)\,db_k^2]\,db_k^2 \quad + \quad \bar{c}(b_k). \tag{C8}$$

Since $k_1, k_2, \bar{k}_1, \bar{k}_2$ are arbitrary and $S_k(.)$ must satisfy (C4) and (C7), $S_k(.)$ can be written in the following for some arbitrary functions $u_k(b), v_k^1(b), and\, v_k^2(b)$:

$$S_k(b,b_k) = u_k(b) - v_k^1(b)[b_k^1 - T_1(b_k)] - v_k^2(b)[b_k^2 - T_2(b_k)] + \bar{C}(b_k). \tag{C9}$$

Given that (C9) must satisfy (C5) and (C7), we must have

$$S_k(b,b_k) = u(b_k) - \frac{\partial u(b_k)}{\partial b_k^1}[b_k^1 - T_1(b_k)] - \frac{\partial u(b_k)}{\partial b_k^2}[b_k^2 - T_2(b_k)] + \bar{C}(b_k) \ .$$

The fact that the reward function is accurate implies that $S_k(.)$ is strongly quasiconcave in outcome b_k. Therefore we must select $u(.)$ such that $S_k(.)$ is strongly quasiconcave in outcome b_k. To see that the bonus function in (13) is proper simply use the rationality boundary conditions and solve for the constant $\bar{C}(b_k)$. Also note that the resulting bonus is responsive.

Q.E.D.

<u>Proof of Theorem 3</u>

Proof of this theorem is straightforward.

$$E_{F_k}S_k(b, B_k) = c\{2b_k^1 E_{F_k}[T_1(B_k)] - (b_k^1)^2 + 2b_k^2 E_{F_k}[T_2(B_k)] - (b_k^2)^2\}$$

Solving $\frac{\partial}{\partial b_k^1}\{E_{F_k}S_k(b,B_k)\} = 0$ and $\frac{\partial}{\partial b_k^2}\{E_{F_k}S_k(b,B_k)\} = 0$ yields

$b_k^1 = E_{F_k}[T_1(B_k)],\ b_k^2 = E_{F_k}[T_2(B_k)]$. Therefore, the bonus function is revealing. Also note that $\frac{\partial^2}{\partial\{b_k^1\}^2}\{E_{F_k}S_k(b,B_k)\} < 0$, and $\frac{\partial^2}{\partial\{b_k^2\}^2}\{E_{F_k}S_k(b,B_k)\} < 0$. This implies that reporting the expected values truthfully maximizes the expected payoff of MM k. The condition $\frac{\partial^2}{\partial\{b_k^1\}^2}\{E_{F_k}S_k(b,B_k)\} < 0$ implies that $S_k(b,b_k)$ is strictly concave in outcome. Hence, it is strongly quasiconcave. Therefore, it is accurate. The bonus is rational as $S_k(b,b_k) = 0$, when $b_k^1 = b_k^2 = 0$. Note also that $S_k(b,b_k)$ is responsive. Therefore it is proper.

Q.E.D.

<u>Proof of Theorem 4</u>

Proof of this theorem involves taking the first-order necessary condition of the utility maximization problem of PM i: $\underset{a_i^1, a_i^2}{Max}\ E_{G_i}R_i(a,Z_i)$, where $Z_i \sim N(n_i, s_i^2)$. This is simple and hence is omitted. Note that $\frac{\partial^2}{\partial\{a_i^1\}^2}\{E\,R_i(a,Z_i)\} < 0$ and $\frac{\partial^2}{\partial\{a_i^2\}^2}\{E\,R_i(a,Z_i)\} < 0$.

Q.E.D.

<u>Proof of Corollary 4</u>

As per our formulation, the payoff of a MM is only the bonus. Therefore, a bonus is always required to eliminate the strategic behavior of a MM. Thus, a bonus is always required. The proof for the product managers is by construction. We show that for a PM, the modified lexicographic allocation rule given by equation (21) is truthful without a bonus. In absence of a bonus, the first order necessary condition of utility maximization problem of PM i (21)-(22) gets modified as follows (note that $y_i \neq 0$):

$$\frac{\partial \ell_i(a)}{\partial a_i^1}[(h_i+p_i+e_i)F[\frac{\ell_i(a)-n_i}{s_i}] - (p_i+e_i)] = 0, \tag{C10}$$

$$\frac{\partial \ell_i(a)}{\partial a_i^2}[(h_i+p_i+e_i)F[\frac{\ell_i(a)-n_i}{s_i}] - (p_i+e_i)] = 0. \tag{C11}$$

We will show that under modified lexicographic rule given by (21), the solution to equations (C10)-(C11) are $a_i^1 = n_i$, $a_i^2 = s_i^2 + n_i^2$; i.e., the PM i reports truthfully. Note that $\frac{\partial \ell_i(a)}{\partial a_i^1}$ = $\frac{\partial \ell_i(a)}{\partial a_i^2}$ = 0 when PM i's allocation is independent of his order (this happens when he is either denied capacity or is given the leftover capacity after allocating capacity up to the rank i-1). Otherwise,

$$\frac{\partial \ell_i(a)}{\partial a_i^1} = 1 - \frac{a_i^1}{[a_i^2 - (a_i^1)^2]^{1/2}} F^{-1}\left(\frac{p_i + e_i}{h_i + p_i + e_i}\right), \text{ and} \tag{C12}$$

$$\frac{\partial \ell_i(a)}{\partial a_i^2} = \frac{1}{2[a_i^2 - (a_i^1)^2]^{1/2}} F^{-1}\left(\frac{p_i + e_i}{h_i + p_i + e_i}\right) . \tag{C13}$$

Substituting (24), (C12) and (C13) into the equations (C10)-(C11) and writing $a_i = F^{-1}\left(\frac{p_i + e_i}{h_i + p_i + e_i}\right)$, we obtain the following first order conditions for PM i:

$$\left[1 - \frac{a_i^1}{[a_i^2 - (a_i^1)^2]^{1/2}} a_i\right] \left\{(h_i + p_i + e_i) F\left(\frac{a_i^1 + a_i [a_i^2 - (a_i^1)^2]^{1/2} - n_i}{s_i}\right) - (p_i + e_i)\right\} = 0, \tag{C14}$$

$$\left[\frac{1}{2[a_i^2 - (a_i^1)^2]^{1/2}} a_i\right] \left\{(h_i + p_i + e_i) F\left(\frac{a_i^1 + a_i [a_i^2 - (a_i^1)^2]^{1/2} - n_i}{s_i}\right) - (p_i + e_i)\right\} = 0. \tag{C15}$$

The solution to (C14)-(C15) is $a_i^1 = n_i$, $a_i^2 = s_i^2 + n_i^2$, which means that modified lexicographic allocation is truthful without bonus. To see that $a_i^1 = n_i$, $a_i^2 = s_i^2 + n_i^2$ solves the system of equations (C14)-(C15), note the following:

(1) $a_i^1 = n_i$, $a_i^2 = s_i^2 + n_i^2$ satisfies the system (C14)-(C15). Therefore, it is a solution.

(2) From (C15) we must have the term in {} equal zero since the term in [ ] is positive.

(3) F is monotonic and both $a_i^1, a_i^2 > 0$. Therefore only one value of the argument of F will satisfy (C15). This means that the positive solution of (C14)-(C15) is unique. Therefore, $a_i^1 = n_i$, $a_i^2 = s_i^2 + n_i^2$ is the solution for (C14)-(C15). **Q.E.D.**

<u>Proof of Corollary 5</u>

The proof is by construction. We provide an example of the implementation of a linear allocation mechanism (which is IR) by an appropriate choice of bonus function. In a linear allocation mechanism the managers are ranked according to a decreasing order of forecasts. The allocation of a manager with rank i is given by:

$$\ell_i(a) = \begin{cases} a_i^1 - \frac{1}{\tilde{n}}\left(\sum_{j=1}^{\tilde{n}} a_j^1 - K\right), & i \le \tilde{n} \\ 0, & i > \tilde{n} \end{cases} \quad \text{(C16)}$$

where, $\tilde{n}$ is the largest integer less than or equal to n such that $\ell_{\tilde{n}}(a) > 0 \; and \; \ell_{\tilde{n}+1}(a) \le 0$. Along with the allocation mechanism of (C9), consider the following reward function:

$$r_i(a, z_i) = c_{1i} a_i^1 - c_{2i}(a_i^1 - z_i)^2 + c_{3i}\{2a_i^2 (z_i)^2 - (a_i^2)^2\}, \forall \text{i}, \quad \text{(C17)}$$

where $c_{1i} = \frac{\tilde{n}-1}{\tilde{n}} y_i \{(h_i + p_i + e_i) F\left(\frac{-(\sum_{j=1}^{\tilde{n}} a_j^1 - K)}{\tilde{n} s_i}\right) - (p_i + e_i)\}$, $c_{2i} >> c_{1i}$, $\forall$i. (C18)

From (C16) and (C17), the solution to the first-order conditions with respect to a_i^2 for the utility maximization problem of PM i is given by:

$$a_i^2 = E(Z_i^2) = s_i^2 + n_i^2$$

With (C16), (C17) and (C18), it can be shown that the first-order conditions with respect to a_i^1 for the utility maximization problem of PM i receiving a positive allocation are given by:

$$F\left(\frac{a_i^1 - \frac{1}{\tilde{n}}(\sum_{j=1}^{\tilde{n}} a_j^1 - K) - n_i}{s_i}\right) = \frac{1}{y_i (h_i + p_i + e_i)} [\{c_{1i} - 2c_{2i}(a_i - n_i)\}\frac{\tilde{n}}{\tilde{n}-1} + (p_i + e_i)]. \quad \text{(C19)}$$

To see that $a_i^1 = n_i$ is the unique solution to (C19), note that the LHS of (C19) is increasing in a_i^1 and RHS of (C19) is decreasing in a_i^1 and $a_i^1 = n_i$ satisfies (C19). Thus, it is the unique solution.

When the PM i is not receiving any positive allocation of product, the condition $c_{2i} >> c_{1i} / 2$ ensures that the first order condition of the PM is satisfied at $a_i^1 = n_i$. **Q.E.D.**

<u>Proof of Corollary 5a</u>

When an allocation rule is a function of the first moment only, we must have $\frac{\partial \ell_i(a)}{\partial a_i^2} = 0$. Now look at the first-order conditions of the utility maximization problem of PM i defined by (21) and (22). When $\frac{\partial \ell_i(a)}{\partial a_i^2} = 0$, equation (22) becomes

$$\int_{-\infty}^{\infty} \frac{\P r_i(a, z_i)}{\P a_i^2} e^{-(z_i - n_i)^2 / 2s^2} dz_i = 0. \qquad \text{(C20).}$$

When an allocation rule is independent of a_i^2, the only reason a PM will report the second moment truthfully is to satisfy equation (C20). However, if a bonus is not provided the PM does not have any incentive to report the second moment truthfully. **Q.E.D.**

<u>Proof of Corollary 6</u>

We want to show that the allocation rule (26) together with bonus function (27) will induce truthful reporting by PM i, i.e. $a_i^1 = n_i$, $a_i^2 = s_i^2 + n_i^2$. This will allocate newsboy optimal quantity to each PM. For simplicity of notation, let us write $q_i = F^{-1}\left(\frac{(1-y_i)(p_i + e_i) - l}{(1-y_i)(h_i + p_i + e_i)}\right)$.

With this notation we have

$$\frac{\partial \ell_i(a)}{\partial a_i^1} = 1 - \frac{a_i^1}{[a_i^2 - (a_i^1)^2]^{1/2}} q_i \text{, and} \qquad \text{(C21)}$$

$$\frac{\partial \ell_i(a)}{\partial a_i^2} = \frac{1}{2[a_i^2 - (a_i^1)^2]^{1/2}} q_i . \qquad \text{(C22)}$$

Substituting (26), (27), (C21) and (C22) into (21) and (22) we obtain the following first-order conditions corresponding to the utility maximization problem of PM i:

$$[-1+\frac{a_i^1}{[a_i^2-(a_i^1)^2]^{1/2}}q_i]$$

$$\{l\frac{y_i}{1-y_i}-y_i(p_i+e_i)+y_i(h_i+p_i+e_i)F(\frac{a_i^1+q_i[a_i^2-(a_i^1)^2]^{1/2}-n_i}{s_i})\}=0, \tag{C23}$$

$$[-\frac{1}{2[a_i^2-(a_i^1)^2]^{1/2}}q_i]$$

$$\{l\frac{y_i}{1-y_i}-y_i(p_i+e_i)+y_i(h_i+p_i+e_i)F(\frac{a_i^1+q_i[a_i^2-(a_i^1)^2]^{1/2}-n_i}{s_i})\}=0. \tag{C24}$$

The solution to (C23)-(C24) is $a_i^1=n_i, a_i^2=s_i^2+n_i^2$, which means that the allocation rule of (26) and bonus of (27) allocate the newsboy optimal quantity to each product manager. This means that an optimal allocation can be implemented by revelation under partial information for multiple statistics reporting. To see that $a_i^1=n_i, a_i^2=s_i^2+n_i^2$ solves the system of equations (C23)-(C24) note the following:

(1) $a_i^1=n_i, a_i^2=s_i^2+n_i^2$ satisfies the system (C23)-(C24). Therefore it is a solution.

(2) From (C24) we must have the term in {} equal zero since the term in [] is positive.

(3) F is monotonic and both $a_i^1, a_i^2>0$. Therefore only one value of the argument of F will satisfy (C24). This means that the positive solution of (C23)-(C24) is unique. Therefore, $a_i^1=n_i, a_i^2=s_i^2+n_i^2$ is the solution for (C23)-(C24). Also note that if the central coordinator chooses to implement an optimal allocation without a bonus, the first-order conditions corresponding to the utility maximization problem of PM i will be given by:

$$[-1+\frac{a_i^1}{[a_i^2-(a_i^1)^2]^{1/2}}q_i]$$

$$\{-y_i(p_i+e_i)+y_i(h_i+p_i+e_i)F(\frac{a_i^1+q_i[a_i^2-(a_i^1)^2]^{1/2}-n_i}{s_i})\}=0, \tag{C25}$$

$$[-\frac{1}{2[a_i^2-(a_i^1)^2]^{1/2}}q_i]$$
$$\{-y_i(p_i+e_i)+y_i(h_i+p_i+e_i)F(\frac{a_i^1+q_i[a_i^2-(a_i^1)^2]^{1/2}-n_i}{s_i})\}=0. \tag{C26}$$

Clearly, $a_i^1=n_i, a_i^2=s_i^2+n_i^2$ is not a solution to (C25)-(C26) for $l>0$. Therefore, a bonus will be required for implementing an optimal allocation by revelation. **Q.E.D.**

Proof of Corollary 6a

Implementing an optimal allocation by revelation means that we must have $a_i^1=n_i, a_i^2=s_i^2+n_i^2$ for each PM when the allocation rule is defined by (26). Note that both mean and standard deviation are unknown to the central coordinator in multiple statistics reporting. Therefore, the allocation rule must be of the form defined by (26). Now look at the first-order conditions of the utility maximization problem of PM i defined by (21) and (22). We have re-written those in equations (C27) and (C28).

$$\underbrace{\int_{-\infty}^{\infty}\frac{\P r_i(a,z_i)}{\P a_i^1}e^{-(z_i-n_i)^2/2s^2}dz_i}_{Term\ 1}-\underbrace{\frac{\P \ell_i(a)}{\P a_i^1}y_i[(h_i+p_i+e_i)F[\frac{\ell_i(a)-n_i}{s_i}]-(p_i+e_i)]}_{Term\ 2}=0 \tag{C27}$$

$$\underbrace{\int_{-\infty}^{\infty}\frac{\P r_i(a,z_i)}{\P a_i^2}e^{-(z_i-n_i)^2/2s^2}dz_i}_{Term\ 1}-\underbrace{\frac{\P \ell_i(a)}{\P a_i^2}y_i[(h_i+p_i+e_i)F[\frac{\ell_i(a)-n_i}{s_i}]-(p_i+e_i)]}_{Term\ 2}=0 \tag{C28}$$

Note that when a bonus function *ri(.)* is proper, term 1 in both (C27) and (C28) equals zero. Therefore, in order to implement an optimal allocation by revelation, term 2 in (C27) and (C28) must equal zero under the allocation rule (26) with $a_i^1=n_i, a_i^2=s_i^2+n_i^2$. However, we can verify by direct substitution that this is not the case. In fact, term 2 equals ly_i which is strictly positive given $y_i>0$ and there exists capacity shortage in the system. Therefore we cannot have term 1 equals to zero in (C27) and (C28). This means that the bonus function $r_i(.)$ is *not* proper.

Q.E.D.

Proof of Theorem B1:

The proof is in line with that of Theorem 1.2 of Fudenberg and Tirole (1991). The proof of Theorem 5(a) similar to that of Theorem B1(a) of MH except that we need payoff functions $\bar{S}_k(.)$ and $\bar{R}_i(.)$ to be quasiconcave in (b_k^1, b_k^2) and (a_i^1, a_i^2), respectively. Making the bonus accurate ensures that they are quasiconcave in b_k^1 and a_i^1, respectively. The fact that $\bar{S}_k(.)$ and $\bar{R}_i(.)$ are quasiconcave in b_k^2 and a_i^2, respectively, follows from the statement of the theorem. The proof of Theorem B1(b) similar to that of Theorem B1(b) in MH. **Q.E.D.**

Proof of Corollary B2

We have shown in the proof of Corollary 5 that an IR allocation can be implemented by revelation in dominant equilibrium. Therefore, it can be implemented by revelation in Bayesian equilibrium. We need only to show that a bonus is required for its implementation by revelation. The proof is by contradiction. Assume that the center does not offer any bonus. Under the assumption of capacity shortage, there exists at least one PM, say PM i, who can increase his expected payoff (under an IR allocation) by reporting $a_i^1 = n_i + e$, instead of $a_i^1 = n_i$ as he receives less than his optimal amount. Therefore, he will report $n_i + e$, instead of n_i. Thus, IR allocation is manipulable without a bonus. **Q.E.D.**

Proof of Corollary B3

We have shown in the proof of Corollary 6 that an optimal allocation can be implemented by revelation in dominant equilibrium. Therefore, it can be implemented by revelation in Bayesian equilibrium. We need only to show that a bonus is required for its implementation by revelation. The proof is by contradiction. Assume that the center does not offer any bonus. Under such condition, the FOC of the utility maximization problem of PM i is given by:

$$\int_{W_{-i}} \{ -\frac{\partial \ell_i(a)}{\partial a_i^1} y_i [(h_i + p_i + e_i) F[\frac{\ell_i(a) - n_i}{s_i}] - (p_i + e_i)\} d k_i(a_{-i}) = 0, \qquad \text{(C29)}$$

$$\int_{W_{-i}} \{ -\frac{\partial \ell_i(a)}{\partial a_i^2} y_i [(h_i + p_i + e_i) F[\frac{\ell_i(a) - n_i}{s_i}] - (p_i + e_i)\} d k_i(a_{-i}) = 0. \qquad \text{(C30)}$$

optimal allocation, can be implemented by revelation when the allocation rule is given by (26). Substitute (26) into (C29)-(C30) and put $a_i^1 = n_i$, $a_i^2 = s_i^2 + n_i^2$ (this ensure implementation by revelation) to obtain

$$\int_{W_{-i}} l\, dk_i(a_{-i}) = 0\,. \tag{C31}$$

Clearly, (C31) cannot be true when there is a capacity shortage in the system. Therefore the FOC will not be satisfied in absence of a bonus. **Q.E.D.**