Service Management – Midterm Key – Spring 2008

1. Assumptions

- Average wait time should be no more than 5 seconds, or 0.0833333 minutes.
- Average branch arrival rates are 30 calls per hour, or 0.5 calls every minute. There are 4 branches.
- Average call center arrival rates are 120 per hour, or 2 calls every minute.
- A branch or call center employee can process 10 calls per hour, or .166667 calls every minute.
- A branch employee costs $20 per hour. A call center employee costs $25 per hour.
- Negative exponential distributions are assumed for processing and arrival times.

(a) Assume λ=0.5 , μ=0.166667, $CV_{arrivals}$=1, $CV_{processing}$=1. The average wait time for 6 branch workers is 0.198286 of a minute, or 11.9 seconds. The average wait time for 7 branch workers is 0.056468 of a minute, or 3.4 seconds. Hiring 7 workers per branch will keep wait times under 5 seconds, on average. The cost per branch is 7*$20 = $140 per hour. The total per hour cost of all four branches is 4*$140 = $560 per hour.

(b) Assume λ=2 , μ=0.166667, $CV_{arrivals}$=1, $CV_{processing}$=1. The average wait time for 17 call center employees is 0.151956 of a minute, or 9.1 seconds. The average wait time for 18 call center employees is 0.075615 of a minute, or 4.5 seconds. The hourly cost for the 18 employees is therefore 18*$25 = $450 per hour. The call center has a lower hourly cost than the branches, so ceteris paribus, the call center is preferred.

(c) Assumptions

- Average wait time for a generic good (calls and emails) should be no more than 5 seconds, or 0.0833333 minutes.
- For each branch, 30 calls arrive per hour and 5 emails arrive per hour.
- For the call center, 120 calls will arrive per hour along with 20 emails.
- Each call center employee can answer 10 calls per hour, or 5 emails per hour. This means the average service time for a call is 6 minutes, and the average service time for an email is 12 minutes.
- Negative exponential distributions are assumed for processing and arrival times.

35 generic goods are arriving per hour, or 0.583333 per minute. (λ=0.583333) Of these generic goods that arrive (5/35) = 14.3% of the time, the generic good will be an email, and (30/35) = 85.7% of the time, the generic good will be a call. The expected service time will therefore be (5/35)*12 + (30/35)*6 = 6.857 minutes per service. Equivalently, 0.1458333 generic goods can be processed each minute. (μ=0.1458333).

The coefficient of variation for arrival times is 1, since we are assuming a negative exponential inter-arrival distribution for the generic good (mean = st.dev. = $1/\lambda$=1.7 minutes).

Assume goods X and Y are two products with random processing times. The average processing times for goods X and Y are E[X] and E[Y], respectively. Similarly, the standard deviation of processing times are σ_X and σ_Y, respectively. A generic variable is made with "p" probability of being good X and (1-p) probability of being good Y. The approximation of variance for this mixed good can be determined by the formula,

$$Var(p \cdot X + (1-p) \cdot Y) = p \cdot \sigma_X^2 + (1-p) \cdot \sigma_Y^2 + p \cdot (1-p) \cdot [E[X] - E[Y]]^2$$

and the squared coefficient of variation for the can be calculated by,

$$CV_{XY}^2 = \frac{Var(p \cdot X + (1-p) \cdot Y)}{(p \cdot E[X] + (1-p) \cdot E[Y])^2}$$

Given the information, E[X]= σ_X = 6 min., E[Y] = σ_Y =12 min., p=(30/35), and (1-p) =(5/35). Given this information, the squared coefficient of variation of the generic good will equal 1.1875.

In summary, λ=0.583333, μ=0.1458333, $CV^2_{arrivals}$ = 1, CV^2_{XY}=1.1875. Therefore, if each branch hired 8 workers, the average wait will be 0.110707 of a minute, or 6.6 seconds. If each branch hired 9 workers, the average wait time will be 0.035637 of a minute, or 2.1 seconds. Thus, the total number of workers to hire per branch will be 9, with an hourly cost of 9*$20=$180 per hour. The total cost across all four branches will be 4*$180=$720 per hour.

If one call center were used, then the arrival rate will be 140 generic goods per hour, or 2.333333 generic goods per minute. Note that the relative probabilities of the generic goods are independent of the call center or branch decision [(20/140) = (5/35) and (120/140) = (5/35)], so the previous estimates on service times remain the same. Thus, λ=2.333333, μ=0.1458333, $CV^2_{arrivals}$ = 1, CV^2_{XY}=1.1875. Hiring 22 workers will have an average wait time of 0.138637 of a minute, or 8.3 seconds. Hiring 23 workers will have an average wait time of 0.07484 of a minute, or 4.5 seconds. The optimal hourly cost of the call center is 23*$25 = $575 per hour. Ceteris paribus, it is cheaper to use the call center rather than the system of four branches.

(d) Assumptions

- Average wait time for a generic good (calls and emails) should be no more than 5 seconds, or 0.0833333 minutes.
- For each branch, 20 calls arrive per hour and 5 emails arrive per hour.
- For the call center, 80 calls will arrive per hour along with 20 emails.
- Each call center employee can answer 10 calls per hour, or 5 emails per hour. This means the average service time for a call is 6 minutes, and the average service time for an email is 12 minutes.
- Negative exponential distributions are assumed for processing and arrival times.

Each branch will have 25 generic goods arriving each hour, or 0.416667 generic goods per minute (λ =0.416667). The squared coefficient of variation for arrival times will be 1. (20/25) = 80% of the time, this generic good will be a call, and (5/25) = 20% of the time, this generic good will be an email. Therefore, the expected processing time is .8*6 min. + .2*12min. = 7.2 minutes. This means that 0.138888 generic goods can be processed per minute (μ =0.1388888). Using the formula in part (c) above, the squared coefficient of variation for processing times will be 1.22222.

From above, λ =0.4166667, μ =0.1388888, $CV^2_{arrivals}$ = 1, CV^2_{XY}=1.22222. If 6 employees are staffed, the average wait time will be 0.264382 of a minute, or 15.9 seconds. If 7 employees are staffed, the average wait time will be 0.075289 of a minute, or 4.5 seconds. Therefore, the total hourly cost of each branch will be 7*$20 = $140, and the total hourly cost of all four branches will be 4*$140= $560 per hour.

The call center will have 100 generic goods arriving each hour, or 1.66667 goods arriving each minute. The remaining parameters have not changed. Therefore, λ =1.66667, μ =0.1388888, $CV^2_{arrivals}$ = 1, CV^2_{XY}=1.22222. If 18 employees were hired, the average wait time will be 0.10082 of a minute, or 6 seconds. If 19 employees were hired, the average wait time will be 0.049739 of a minute, or 3 seconds. Thus, the hourly cost of the call center will be 19*$25 = $475 per hour. Ceteris paribus, the call center is still cheaper than the four branches when planning for the typical hour.

Assuming that 7 employees are staffed in each branch, 30 calls were arriving each hour, and 5 emails were arriving each hour, then λ =0.583333, μ =0.1458333, $CV^2_{arrivals}$ = 1, CV^2_{XY}=1.1875, and s=7. The average wait time in the queue will be 0.337775 of a minute, or 20.3 seconds. The probability that one will have to wait is 0.135110, or 13.5%.

Assuming that 19 employees were staffed in the call center, 120 calls were arriving each hour, and 20 emails were arriving each hour, then λ =2.333333, μ =0.1458333, $CV^2_{arrivals}$ = 1, CV^2_{XY}=1.1875, and s=19. The average wait time in the queue will be 0.933922 of a minute, or 56 seconds. The probability that one will have to wait is 0.373569, or 37.4%.

2. a)

	Area				
	1	2	3	lambda	3
Travel time to Store A	5	10	12		
Travel time to Store B	25	16	9		
Number of Households	500	600	460		
Expenditures per household	300	300	300		

		Area		
	Size	1	2	3
Attraction to A	1.5	0.012	0.0015	0.000868
Attraction to B	1	0.000064	0.000244	0.001372

	Area		
	1	2	3
Probability of using Store A	0.994695	0.860022	0.38756
Probability of using Store B	0.005305	0.139978	0.61244

	Area			
	1	2	3	All Areas
Total Expenditures Store A	$149204.20	$154804.00	$53483.25	$357491.50
Total Expenditures Store B	$795.75	$25195.97	$84516.75	$110508.50

Market Share A	76.4%
Market Share B	23.6%

3. a)

Many answers are possible. The best answers emphasize that the service package contains not only food, but convenience and reduced time costs on the students.

(b)

Many answers are possible. Diagrams should include all stages of the operations from the kitchen to the customer. This process might include stages such as ordering, preparation, quality control, delivery, storage, customer transactions, and disposal. The best answers likely emphasize that the delivery stage would cause a bottleneck, and that bottlenecks cause quality of foods to deteriorate.

(c)

Many answers are possible, and depend partly on the answer provided in part (b). Answers must be logical and emphasize the lessons learned in class.