



BUAD 467/667
Service Management
Spring 2008

Professor Patrick T. Harker

Class 10a
Performance Measurement II

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Outline for the Class

- Linear Programming: Introduction by example
- DEA: The Basics
- DEA: The Mathematics
- DEA: Burger Palace Example
- Analyzing Service Delivery in Banking
- Next Class

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Linear Programming: Basics

- Uses a mathematical programming technique to solve a general type of problem.
- Define a linear objective function to optimize (i.e. minimize costs or maximize profits).
- Define the relevant constraints in linear form.
- The simplex algorithm: an iterative procedure that continues to increase (if max.) or decrease (if min.) the objective function until all constraints are binding or until the objective function can no longer be improved.

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Linear Programming: Example

You own a factory that makes Tables (T) and Chairs (C).

Each table sells for \$50 and each chair sells for \$40.

Each table and chair must go through three processes.

Process	Table	Chair	Capacity
Lathe	4 hrs	4 hrs	560 hours
Assembly	3 hrs	2 hrs	400 hours
Finishing	2 hrs	4 hrs	400 hours

Your objective? → Maximize Profit

Your constraints? → Your capacity (sunk costs)

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Linear Programming: The Setup

Maximize $X = \$50 \cdot T + \$40 \cdot C$

Subject to $4 \cdot T + 4 \cdot C \leq 560$ (Lathe constraint)

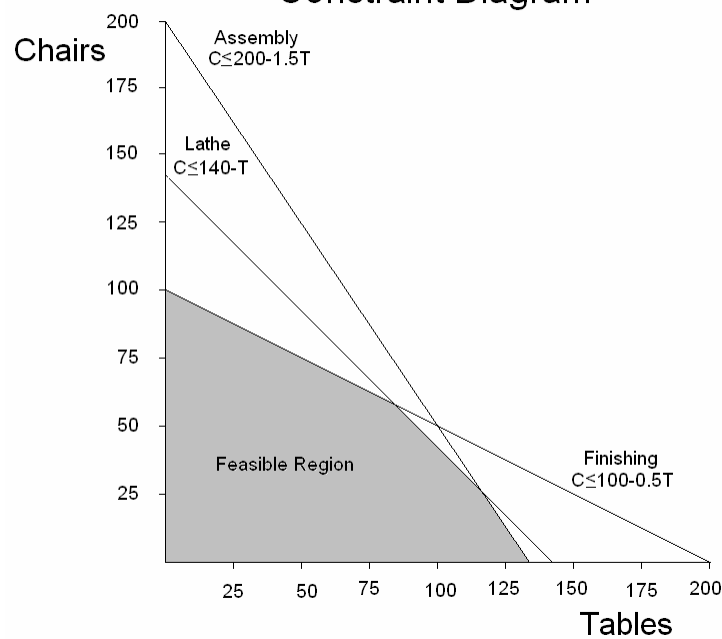
$3 \cdot T + 2 \cdot C \leq 400$ (Assembly constraint)

$2 \cdot T + 4 \cdot C \leq 400$ (Finishing constraint)

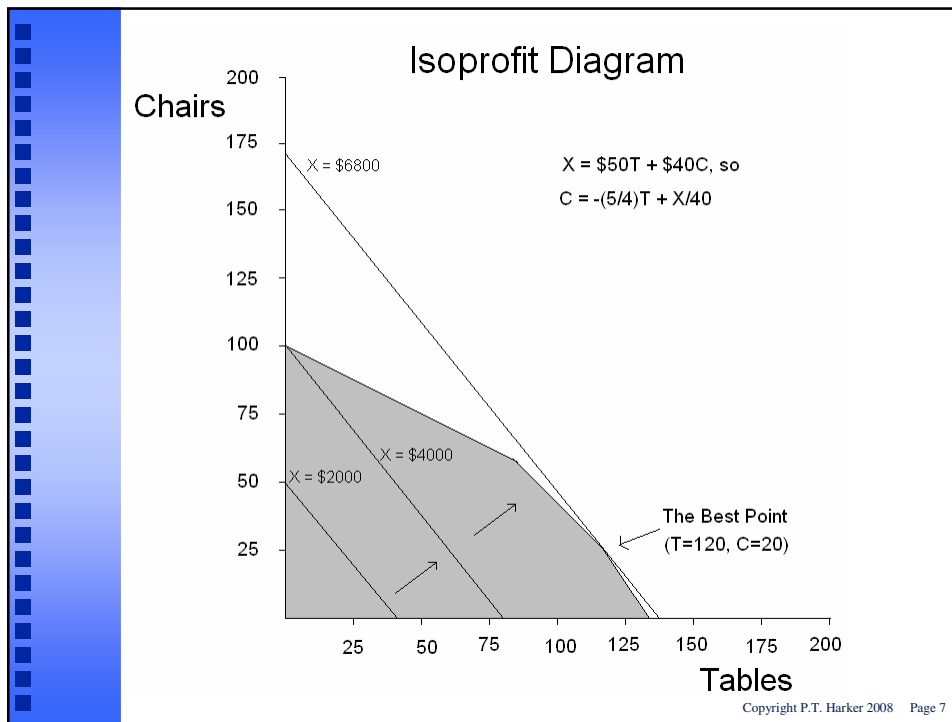
$T \geq 0$ and $C \geq 0$ (Logic constraint)

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Constraint Diagram



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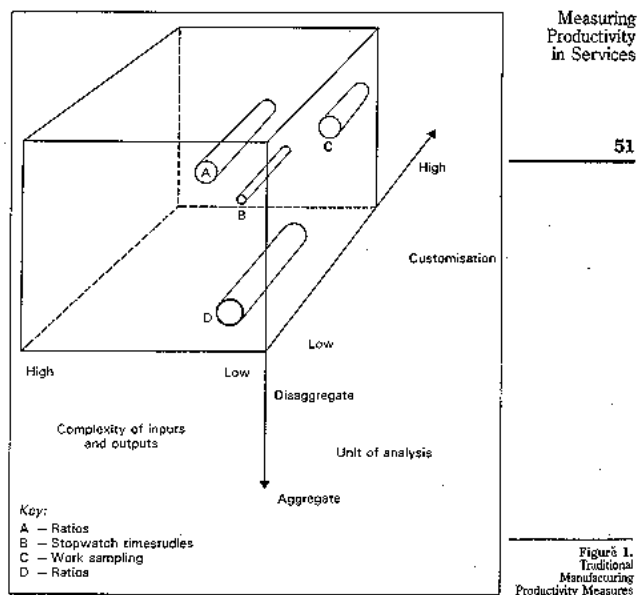
General Ideas of Linear Programming

- If a solution exists, there will be a finite number of corners in the feasible region.
- If an optimal solution exists, it can always be found on a corner in the feasible region.
- The simplex algorithm searches each corner systematically to guarantee an optimal solution will be found, provided one exists.
- The simplex algorithm can be modified to control for constraints of equality.
- This is the underlying algorithm that enables DEA.

DEA: The Basics

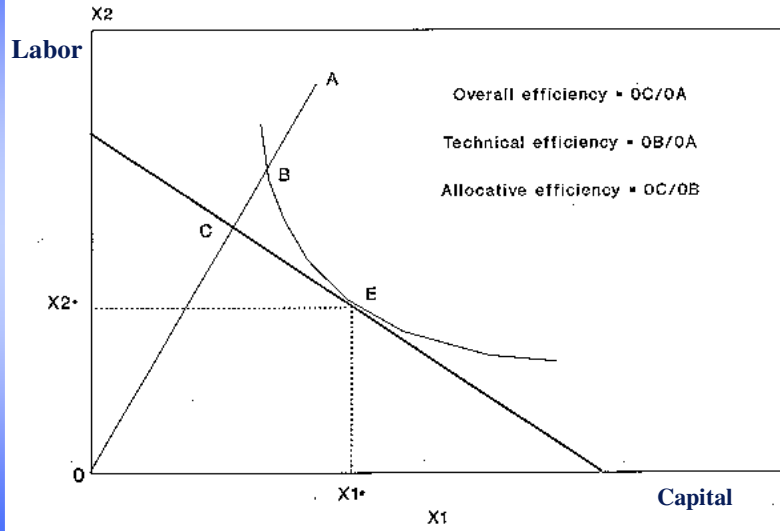
- the service advantage: multiple **decision making units**
- Banc One and benchmarking: their key to improving bank efficiency
- once you collect benchmarking data, what do you do with it?
- more basic, who do you benchmark?

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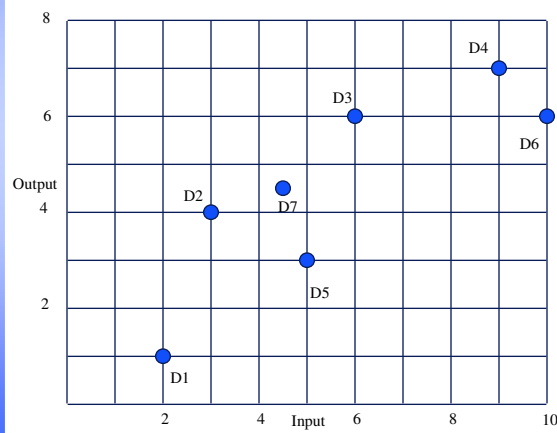
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MEASURING EFFICIENCY



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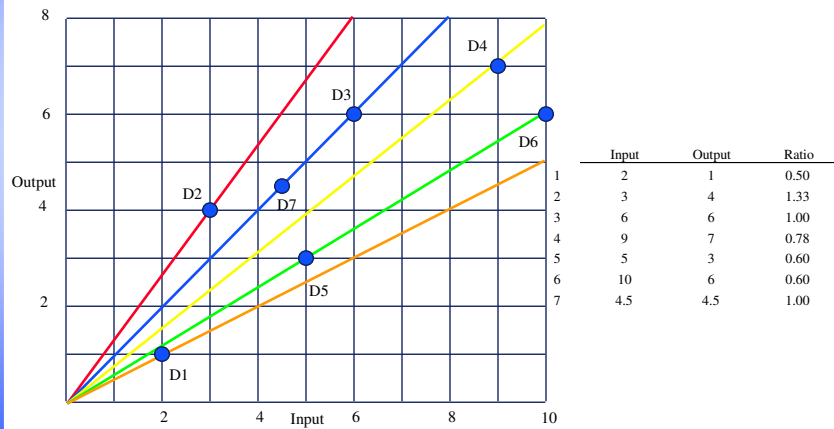
Graphical Interpretation of Efficiency



	Input	Output
1	2	1
2	3	4
3	6	6
4	9	7
5	5	3
6	10	6
7	4.5	4.5

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Traditional Efficiency



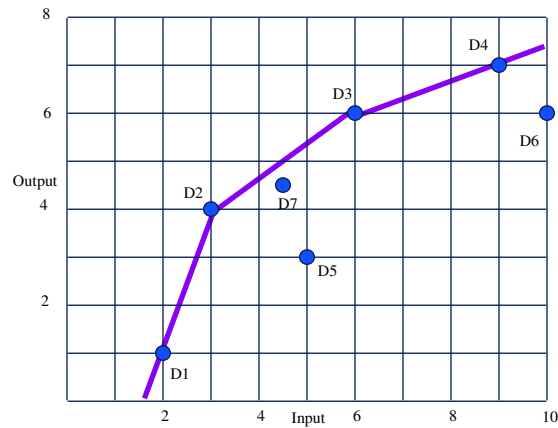
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Problems With Traditional Efficiency Measures

- assumes fixed ratios with multiple inputs and/or outputs
- assumes constant returns to scale
- don't tell you where to look for improvements!

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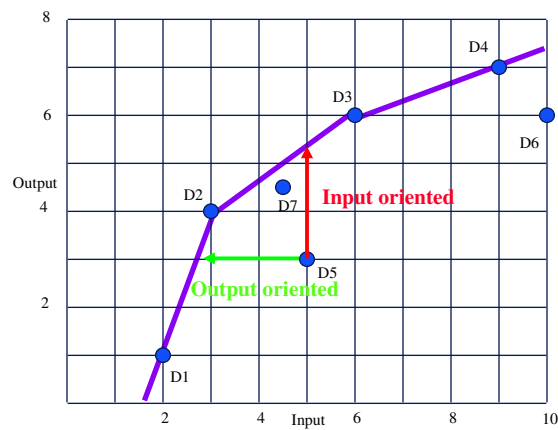
Frontier Efficiency



	Input	Output
1	2	1
2	3	4
3	6	6
4	9	7
5	5	3
6	10	6
7	4.5	4.5

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Frontier Efficiency



	Input	Output
1	2	1
2	3	4
3	6	6
4	9	7
5	5	3
6	10	6
7	4.5	4.5

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DEA: The Mathematics

maximize My Efficiency Score
 s.t. All Scores ≤ 1

by choosing weights for the inputs and outputs.

Efficiency = $\frac{\text{weighted sum of outputs}}{\text{weighted sum of inputs}}$

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$$\begin{aligned} \text{Efficiency for person } k &= \frac{\text{Weighted Sum of Outputs}}{\text{Weighted Sum of Inputs}} \\ &= \frac{U1 \cdot Y1,k + U2 \cdot Y2,k + \dots + Un \cdot Yn,k}{V1 \cdot X1,k + V2 \cdot X2,k + \dots + Vm \cdot Xm,k} \\ &= \frac{\sum_{i=1}^n Ui \cdot Yi,k}{\sum_{j=1}^m Vj \cdot Xj,k} \end{aligned}$$

- Let $Y1,k$ = Output 1 for person k , $Y2,k$ = Output 2 for person k , ... Yn,k = Output n for person k .
- Let $X1,k$ = Input 1 for person k , $X2,k$ = Input 2 for person k , ... Xm,k = Input m for person k .
- $U1$ = Weight of Output 1, $U2$ = Weight of Output 2, etc.
- $V1$ = Weight of Input 1, $V2$ = Weight of Input 2, etc.
- Assume there are 'N' DMU's in the reference set, S.

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Calculating efficiency is straightforward if all the U_i 's and the V_j 's are known. Plug and chug!

$$\text{Efficiency for person } k = \frac{\sum_{i=1}^n U_i \cdot Y_{i,k}}{\sum_{j=1}^m V_j \cdot X_{j,k}}$$

But what if the weights aren't known, aren't easily quantifiable, or change frequently?

DEA makes relative comparisons in a reference set when there is little or no information about the weights of inputs and outputs.

The DMU under analysis is given the benefit of the doubt.

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How DEA Works

Maximize: My Efficiency Score (I'm person k)

Subject to: All Scores ≤ 1

by choosing weights for the inputs and outputs.

$$\begin{aligned} \text{Max. } & \frac{\sum_{i=1}^n U_i \cdot Y_{i,k}}{\sum_{j=1}^m V_j \cdot X_{j,k}} & \text{s.t. } & \frac{\sum_{i=1}^n U_i \cdot Y_{i,s}}{\sum_{j=1}^m V_j \cdot X_{j,s}} \leq 1 \quad \text{for every } s \text{ in } S=(1,2,3,\dots,N) \\ & & & U_i \geq 0, \quad \text{for every } i \\ & & & V_j \geq 0, \quad \text{for every } j \end{aligned}$$

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Input-Oriented Methods:

minimize My Weighted Sum of Inputs
s.t. Weighted Outputs \leq Weighted Inputs
Weighted sum of Outputs = 1

Will generally lead to a different inefficiency result than output oriented methods.

Burger Palace

Service Unit	Meals Sold	Labor hours	Material costs
1	100	2	200
2	100	4	150
3	100	4	100
4	100	6	100
5	100	8	80
6	100	10	50

Unit 1's Problem

$$\begin{aligned} &\text{maximize} && 100U_1 \\ &\text{subject to:} && 100U_1 - 2V_1 - 200V_2 \leq 0 \\ & && 100U_1 - 4V_1 - 150V_2 \leq 0 \\ & && 100U_1 - 4V_1 - 100V_2 \leq 0 \\ & && 100U_1 - 6V_1 - 100V_2 \leq 0 \\ & && 100U_1 - 8V_1 - 80V_2 \leq 0 \\ & && 100U_1 - 10V_1 - 50V_2 \leq 0 \\ & && 2V_1 + 200V_2 = 1 \\ & && U_1, V_1, V_2 \geq 0 \end{aligned}$$

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Graphical Analysis:

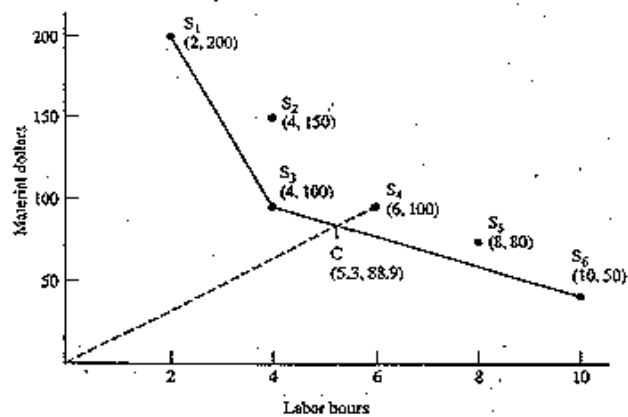


FIGURE 12.8
Productivity frontier for Burger Palace.

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DEA: Burger Palace Example

Burger Palace DEA Example								
Service Unit	Meals Sold	Labor-Hours	Material costs	Sum to One	Limits	Objective	Efficiency	
1	100	2	200	0.999999993		0	0.99999999	100.00%
2	100	4	150	1.166666651	-0.16666666		0.99999999	85.71%
3	100	4	100	0.999999984	8.92367E-09		0.99999999	100.00%
4	100	6	100	1.333333309	-0.33333332		0.99999999	75.00%
5	100	8	80	1.599999968	-0.59999997		0.99999999	62.50%
6	100	10	50	1.833333292	-0.8333333		0.99999999	54.55%
Weights	u1	v1	v2					
	0.01	0.16666666	0.003333333					

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Results for Unit 1

Changing Cells

Cell	Name	Final Value	Reduced Gradient
\$B\$13	u1	0.01	0
\$C\$13	v1	0.166666666	0
\$D\$13	v2	0.003333333	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
\$E\$5	Sum to One	0.999999991	0.999999999
\$F\$5	Limits	1.11022E-16	0.999999941
\$F\$6	Limits	-0.166666667	0
\$F\$7	Limits	-1.63332E-09	0
\$F\$8	Limits	-0.333333333	0
\$F\$9	Limits	-0.599999999	0
\$F\$10	Limits	-0.833333331	0

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Results for Unit 2

Changing Cells

Cell	Name	Final Value	Reduced Gradient
\$B\$13	u1	0.008571419	0
\$C\$13	v1	0.142856976	0
\$D\$13	v2	0.00285714	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
\$E\$6	Sum to One	0.999998833	0.857142856
\$F\$5	Limits	-4.2575E-09	0.285714258
\$F\$6	Limits	-0.142856981	0
\$F\$7	Limits	-5.65749E-09	0.714285734
\$F\$8	Limits	-0.285713959	0
\$F\$9	Limits	-0.514285121	0
\$F\$10	Limits	-0.714284889	0

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Results for Unit 3

Changing Cells

Cell	Name	Final Value	Reduced Gradient
\$B\$13	u1	0.010000008	0
\$C\$13	v1	0.16666681	0
\$D\$13	v2	0.003333336	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
\$E\$7	Sum to One	1.000000857	0.999999929
\$F\$5	Limits	-9.93422E-09	0
\$F\$6	Limits	-0.166666821	0
\$F\$7	Limits	-1.15675E-08	0.999999999
\$F\$8	Limits	-0.333333631	0
\$F\$9	Limits	-0.600000527	0
\$F\$10	Limits	-0.833334061	0

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Results for Unit 4

Changing Cells

Cell	Name	Final Value	Reduced Gradient
\$B\$13	u1	0.008888889	0
\$C\$13	v1	0.055555555	0
\$D\$13	v2	0.006666667	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
\$E\$8	Sum to One	0.999999998	0.888888891
\$F\$5	Limits	-0.555555554	0
\$F\$6	Limits	-0.333333332	0
\$F\$7	Limits	2.08191E-10	0.777777796
\$F\$8	Limits	-0.111111111	0
\$F\$9	Limits	-0.088888889	0
\$F\$10	Limits	1.11022E-16	0.22222222

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If Unit 4 were efficient...

0.778 (Inputs of Unit 3) + 0.222 (Inputs of Unit 6)

=

0.778 (4 labor hours + 0.222 (10 labor hours
\$100) \$50)

= 5.332 labor-hours, \$88.90

versus

6 labor-hours and \$100

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Results for Unit 5

Changing Cells

Cell	Name	Final Value	Reduced Gradient
\$B\$13	u1	0.009090918	0
\$C\$13	v1	0.056818237	0
\$D\$13	v2	0.006818188	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
\$E\$9	Sum to One	1.000000978	0.909090906
\$F\$5	Limits	-0.568182373	0
\$F\$6	Limits	-0.340909424	0
\$F\$7	Limits	5.516E-10	0.454545455
\$F\$8	Limits	-0.113636474	0
\$F\$9	Limits	-0.090909179	0
\$F\$10	Limits	6.77355E-10	0.545454535

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Results for Unit 6

Changing Cells

Cell	Name	Final Value	Reduced Gradient
\$B\$13	u1	0.010000009	0
\$C\$13	v1	0.062500057	0
\$D\$13	v2	0.007500007	0

Constraints

Cell	Name	Final Value	Lagrange Multiplier
\$E\$10	Sum to One	1.000000909	1.000000001
\$F\$5	Limits	-0.625000564	0
\$F\$6	Limits	-0.375000338	0
\$F\$7	Limits	2.09688E-09	0
\$F\$8	Limits	-0.125000112	0
\$F\$9	Limits	-0.100000089	0
\$F\$10	Limits	7.45091E-10	1.000000053

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Graphical Analysis:

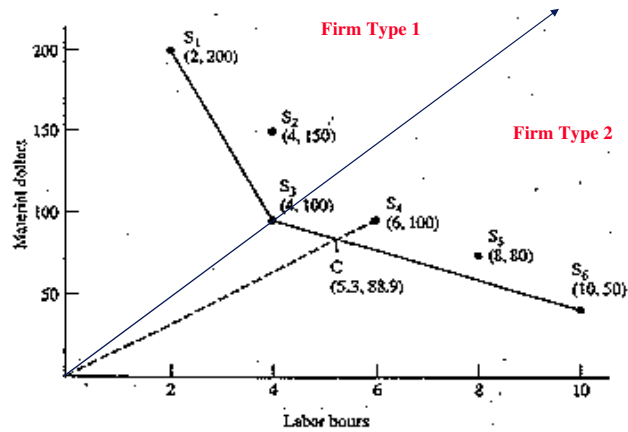


FIGURE 12.6
Productivity frontier for Burger Palace.

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In general...

- can analyze input and output changes
- tells you where to look for changes
- tells you what is possible

- other methods are available as well (variable returns)
- be careful of weights summing to one (zero is a firm).
- defining your inputs and outputs is very important
- beware of measurement error and small samples
- quasi-statistical inference still possible with simulation
- you can put “bounds” on the weights if you have prior knowledge about some variables.
- multiplier vs. envelopment form

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Performance Measurement III

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Outline for the Class

- Performance measurement in practice – Mike Middaugh, UD

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Next Class

- Nashville National Bank Case
- IT in Services

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