Is There a Term Structure of Futures Volatilities?

Reevaluating the Samuelson Hypothesis*

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Abstract

The Samuelson hypothesis implies that the volatility of futures price changes increases as a contract's delivery date nears. In markets where the Samuelson hypothesis holds, accurate valuation of futures-related derivatives requires that a term structure of futures volatilities be estimated. We develop a framework for predicting those markets where the Samuelson hypothesis should be expected to hold. In contrast to a prominent reinterpretation of the hypothesis, we show that clustering of information flows near the delivery date is not a necessary condition. We show instead that the hypothesis will generally be supported in markets where spot price changes include a predictable temporary component, and we argue that this condition is much more likely to be met in markets for real assets than for financial assets. Finally, we provide empirical evidence consistent with our predictions.

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Understanding the determinants and dynamics of futures price volatility is useful for a number of applications. For example, some models (e.g. Hirshleifer (1988)) imply that the equilibrium risk premium in futures markets depends on futures price volatility. Perhaps most importantly, the accurate valuation of options or related derivatives on futures requires knowledge of the dynamic evolution of futures price volatility.

This study focuses on a specific aspect of futures price volatility: the relation between volatility and time until contract expiration. Samuelson (1965) first investigated this relation, advancing the hypothesis that the volatility of futures price changes should increase as the delivery date nears. This prediction is sufficiently well known that it is widely referred to as the "Samuelson Hypothesis". In markets where the Samuelson hypothesis holds, the accurate valuation of options or related derivatives on futures requires that estimates of futures volatility depend on the time remaining until the underlying futures contract matures.

Numerous studies have investigated the Samuelson hypothesis empirically, and the hypothesis has been supported in a subset of markets. Anderson (1985) finds support for the hypothesis in wheat, oats, soybeans, soybean meal, live cattle, and cocoa prices, but not for silver. Milonas (1986) also finds strong support for the hypothesis in agricultural markets, but little or no support when using gold, copper, GNMA, T-bond, and T-bill prices. Grammatikos and Saunders (1986) find no relation between futures return volatility and time-to-maturity for currency futures prices. Despite the mixed empirical evidence, there has to date been little attempt to provide an economic analysis that predicts why the hypothesis should be supported in some markets but not others.

The primary objective of this study is to develop a framework to predict those markets in which the Samuelson hypothesis should be expected to hold, and to predict cross-sectional variation in the strength of the relation between volatility and time to maturity. We begin with a clarification of the economic framework originally used by Samuelson and a critique of a subsequent reinterpretation of the hypothesis. We then provide our analysis of the economic issues, and we make cross-sectional predictions regarding the validity of the hypothesis. Finally, we employ prices from an array of futures markets to test the accuracy of these predictions.

Samuelson's 1965 study is entitled "Proof that Properly Anticipated Prices Fluctuate Randomly". In keeping with the title, each formal proof in the paper is devoted to demonstrating that futures price changes are unpredictable in equilibrium, even if changes in the spot price of the good are partially predictable. [1] Samuelson addressed relations between futures volatility and time to maturity only in the form of an example. He provides neither formal proofs or statements of sufficient or necessary conditions for what has come to be known as the Samuelson hypothesis. The example he uses to illustrate the hypothesis is based on the assumptions that (i) each futures price equals the trading date expectation of the delivery date spot price, and (ii) that the spot price itself follows a first order autoregressive process, with an AR(I) coefficient equal to 0.5. Importantly, this specification implies that the spot price is stationary, and reverts in the long run to a mean of zero.[2]

More recent studies have chosen to reinterpret the Samuelson hypothesis by tying it to a phenomenon not considered by Samuelson: time variation in the rate of information flow. In an influential article, Anderson and Danthine (1983, p. 262) write:

> "We stress that there is no inherent tendency for futures prices to become more volatile as delivery approaches. What matters is when information resolving uncertainty flows into the

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[1] Samuelson initially invokes the assumption of risk neutrality and proves that equilibrium futures price changes are entirely unpredictable. He subsequently generalizes this theorem to allow for risk aversion and an upward trend in futures prices that compensates for risk.

[2] The importance of the stationary spot price assumption was apparently first emphasized by Rutledge (1976). He shows that when the AR(1) coefficient in Samuelson's example is set to 1.0, so that the spot price follows a random walk, the implication that the futures variance increases near the expiration date no longer holds. In his response to Rutledge, Samuel son (1976) rigorously defends the assumption that spot prices are stationary.

> market. Our results show that Samuelson's original proposition is properly viewed as a hypothesis stating that more uncertainty tends to be resolved or more information flows into the market as delivery approaches. "

This reasoning has become prevalent in discussions of the Samuelson hypothesis. For example, in a widely used textbook, Kolb (1991, page 139) writes:

> "While the mathematics of Samuelson's model are somewhat complex, the intuition is clear. Price changes are large when more information is revealed about a commodity. Early in a futures contract's life, little information is known about the future spot price for the underlying commodity. Later, as the contract nears maturity, the rate of information acquisition increases. For example, little is known about a corn harvest a full year before harvest time. As the harvest approaches, the market gets a much better idea of the ultimate price that corn will command. "

The degree to which contemporary discussion of the Samuelson hypothesis diverges from the original analysis is illustrated by comparing the preceding quote to Samuelson's (1965, p. 786) own statement regarding the role of harvest patterns:

> "The present theory can contribute an elegant explanation of why we should expect far-distant futures to move more sluggishly than near ones. Its explanation does not lean at all on the undoubted fact that, during certain pre-harvest periods when stocks are normally low, changes in spot prices can themselves be expected to experience great volatility".

It is certainly possible that a systematic clustering of information flows near futures delivery dates could cause commensurate increases in price change variances at those times, consistent with the Samuelson hypothesis. However, we note the absence of compelling explanations for why information flows should cluster near futures maturity dates. In the case of agricultural futures in particular, contracts mature not only near harvest dates, but throughout the year. More importantly, we assert that time variation in information flows is not a necessary condition for the Samuelson hypothesis, which can be expected to hold in some markets where information flows do not cluster near delivery dates. Therefore, explanations that invoke systematic variation in rates of information flow are unlikely to provide accurate cross-sectional predictions regarding the validity of the Samuelson hypothesis.

In this paper, we present a new analysis of the economic issues underlying the prediction of increasing futures market volatility as contract expiration approaches. We show that neither the clustering of information flows near delivery dates nor the assumption that each futures price is an unbiased forecast of the delivery date spot price are necessary conditions for the success of the hypothesis. We refocus attention on the key condition: the stationarity of spot prices. We show that the Samuelson hypothesis does not require that prices are strictly stationary; the existence of a temporary component in price changes (along with a permanent component such as that introduced by general price level inflation) is sufficient. Finally, our model incorporates the requirement that those who hold inventory positions in the spot asset earn a competitive return. In contrast, the spot price process in Samuelson's example was specified without regard to market clearing conditions in the spot market.

Our analysis predicts that the Samuelson hypothesis will hold only in those markets where spot price changes include a temporary component, so that investors expect some portion of a typical price change to be reversed in the future. Empirical examination of this prediction is complicated by the need to identify the markets that meet this criterion. We rely on principles of competitive equilibrium to identify those markets. The presence of a temporary component in spot price changes implies expected capital losses (or decreases in expected rates of capital gain) following unusual price increases, and vice versa. Since the spot asset must pay a competitive return to induce the holding of inventories, such variation in expected rates of capital gain must in equilibrium be offset by changes in the economic costs of carrying inventory. The no-arbitrage "cost-of-carry" relation implies that the economic cost of carrying inventory is revealed by the slope of the futures term structure (to be formally defined below), which is observable. Combining these insights, our analysis predicts that the Samuelson Hypothesis will be empirically supported in those markets that exhibit negative covariation between spot price changes and changes in the slope of the futures term structure.

Further, we argue that the most plausible reason for substantial time variation in inventory carrying costs derives from the variation of real service flows or “convenience yields”. In particular, positive covariation between convenience yields and spot prices lead to mean reverting spot prices in equilibrium, and is sufficient to support the Samuelson hypothesis. Since financial assets do not provide service flows, we predict that the Samuelson hypothesis will not hold for financial futures.

We present empirical evidence consistent with our predictions. The Samuelson hypothesis is supported in those markets, including agricultural, crude oil, and to a lesser extent, metals, where the covariation between changes in spot prices and the futures term slope is significantly negative. We find no support for the hypothesis in financial futures. These results imply that the accurate valuation of options and related derivatives on real asset futures will require the estimation of a volatility term structure that depends on time to expiration.

I. An Alternate Development of the Samuelson Hypothesis.

In this section, we present a new analysis of the relation between futures price volatility and time to contract expiration. In developing this analysis, we have four main goals. First, we demonstrate that time variation in information flows is not a necessary condition for the Samuelson hypothesis. Second, we avoid invoking the assumption that each futures price is an unbiased forecast of the delivery date spot price, since that assumption has been challenged on both theoretical and empirical grounds.[3] Third, we identify the precise role of mean reverting asset prices for the success of the hypothesis. Fourth, we ensure that the evolution of spot prices is consistent with market-clearing conditions. This analysis is general in that it makes minimal assumptions regarding economic equilibrium, requiring only that the no-arbitrage cost-of-carry condition links spot and futures prices and that the marginal holder of the spot asset earns a competitive return.

[3] See, for example, Hirshleifer (1988), Bessembinder (1992) and the references contained therein.

A. The role of time variation in the futures term slope.

Let P_t denote the date t spot price of a good upon which futures contracts are written and let r_t denote the per period risk-free interest rate at date t. Let c_t denote the net per period cash flow yield accruing to the marginal holder of spot inventory, including any explicit cash payments such as dividends or coupon payments, plus the value of any implicit service flows or "convenience yields", less any cash outflows such as storage or insurance costs. If P_t is to equate supply and demand, the marginal holder of inventory must anticipate a competitive return. The cash flow yield, C_t, comprises a component of the return to holding inventory, while the remainder of the anticipated return accrues in the form of price appreciation. The relation between the equilibrium date t price and the expected spot price at arbitrary future date t +j can be stated as:

$$(1)\quad E_t(P_{t+j}) = P_t\, e^{(r_t + \pi - c_t)j}\ ,$$

where $E_t(.)$ denotes an expectation formed at time t, conditional on the time t information set. Here, π is defined implicitly as a premium in the anticipated rate of price appreciation that is sufficiently large to yield a competitive expected return to holding inventory. Specifying the determinants of this equilibrium risk premium is beyond the scope of this paper; we simply note that a non-zero premium may be required to clear the spot market. For simplicity, we assume for our formal analysis that π is an intertemporal constant. Below, we consider the implications of allowing the risk premium to vary over time.

Let $F_{t,T}$ denote the date t futures price for delivery of the underlying good at date T. We link spot and futures prices by the well-known cost-of-carry relation:

$$(2)\quad F_{t,T} = P_t\, e^{(s_t(T-t))},$$

where $s_t \equiv r_t - c_t$. The cost-of-carry relation is a no-arbitrage condition dating from Working (1949) and Brennan (1958), that links futures prices to contemporaneous spot prices by the per-period cost to the marginal trader of holding the underlying asset in inventory, s_t.[4] Though the economic interpretation of s_t is the cost of carrying inventory, we also refer to s_t as the "futures term slope". This label reflects that, given equation (2), s_t can be observed as the slope of a line linking the log spot price to the log futures price when plotted against time to delivery.

Define $\Delta s_t \equiv s_{t+1} - s_t$ as the change from date t to date t+1 in the futures term slope, $\Delta f_t \equiv \ln(F_{t+1,T}) - \ln(F_{t,T})$ as the change in the log futures price for the contract maturing at date T, $\Delta p_t \equiv \ln(P_{t+1}) - \ln(P_t)$ as the change in the log spot price, and $\tau \equiv (T-t-1)$ as the remaining number of periods until contract expiration. Then, applying (2) for dates t and t+1, the change in the log futures price can be stated as:

(3) $\mathbf{\Delta f_t = \Delta p_t + \Delta s_t \tau - s_t}$.

Next, define $u_t \equiv \ln(P_{t+1}/E_t(P_{t+1}))$ as the unexpected rate of spot price appreciation. Noting that this definition also implies that $\ln(P_{t+1}) = \ln(E_t(P_{t+1})) + u_t$, and using equation (1), the change in the log spot price is:

(4) $\mathbf{\Delta p_t = s_t + \pi + u_t}$.

Using (3) and (4) we have:

(5) $\mathbf{\Delta f_t = \pi + u_t + \Delta s_t \tau}$.

The cost-of-carry relation implies that the change in the log futures price is the sum of three components: the ex ante spot market risk premium, the unexpected rate of spot market price appreciation, and the change in the futures term slope weighted by the remaining time to expiration. Using (5), the variance of futures price changes is:

(6) $\mathbf{VAR(\Delta f_t) = VAR(u_t) + \tau^2 VAR(\Delta s_t) + 2\tau COV(u_t, \Delta s_t)}$.

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[4]It is also useful to note that equations (1) and (2) jointly imply that $E_t(P_T) = F_{t,T}e^{\pi(T-t)}$. The cost-of-carry relation implies that the spot market premium π and the bias in the futures price as a forecast of the delivery date spot price are jointly determined. In particular, if $\pi = 0$ then the futures price is an unbiased forecast, while if π is positive the futures price is a downward biased estimate of the spot price.

where VAR and COV denote variance and covariance, respectively. The Samuelson hypothesis implies that $VAR(\Delta f_t)$ increases as τ, the number of periods until delivery, decreases. Note that if the futures term slope, s_t, is an intertemporal constant, then the last two terms in (6) equal zero. Then, the Samuelson hypothesis holds if and only if the variance of unexpected spot returns increases as the delivery date approaches. Any explanation that excludes time variation in the futures term slope can motivate the Samuelson hypothesis only by motivating increases in spot volatility near the expiration date. Note that a relaxation of equation (1) to allow for time variation in the risk premium π does not alter expression (6), as long as the ex ante premium for the interval t to t+ 1 is known at date t.

Whether spot price change volatility increases as futures delivery dates approach is an empirical question. If, however, spot return variances do increase near each delivery date, then the observed dynamics of futures price change volatility would be relatively complex. The life of a futures contract typically spans the expiration dates of several earlier maturity contracts. Any increase in spot return variances associated with the maturity of the earlier contract would, given a constant futures term slope, also increase the variances of futures prices for contracts with more distant delivery dates. If so, an individual contract would display several periods of high variance during its life, with one high variance period associated with the expiration of each nearer contract. The Samuelson hypothesis does not predict such a "saw-toothed" volatility pattern. Hence, we conclude that an accurate representation of the economics underlying the hypothesis must focus on stochastic variation in the futures term slope rather than increases in spot return variances near delivery dates.

If $\varsigma AP(\Delta S_t)$ is a positive constant, then its contribution to the variance of the futures price change, quantified by the second term on the right side of (6), *increases* with the square of the remaining time to contract expiration, r. Higher variance for more distant delivery dates is inconsistent with the Samuelson hypothesis. The absolute contribution of the third term on the right side of (6) also increases with time to expiration. However, unlike the second term, the sign of the third term's contribution can be positive or negative. If $COV(u_t,\Delta s_t) < 0$ then this third term increases (toward zero) as the trading date t approaches the contract expiration date T, with the rate of increase proportional to twice the remaining time to contract expiration. If, as we argue above, the Samuelson hypothesis does not rely on time variation in $VAR(u_t)$, the hypothesis *must* rely on negative covariation between spot returns and changes in the futures term slope.

B. The Role of Mean Reverting Spot Prices

We next link negative covariation between spot price and futures term slope changes to mean reversion in spot prices. We do so by considering relations between shocks to the current spot price and the revision in expectations regarding the future spot price. We focus on cases where the rate of revision from time t to time t+1 in the expectation of the delivery date spot price, $\ln[E_{t+1}(P_T)/E_t(P_T)]$, is a nonstochastic proportion of the unexpected spot return, $u_t = \ln(P_{t+1}/E_t(P_{t+1}))$. Let $\varepsilon_{t\tau} \equiv \ln[E_{t+1}(P_T)/E_t(P_T)]/u_t$ denote this factor of proportionality. Since the numerator and denominator of $\varepsilon_{t\tau}$ are each stated in proportional terms, $\varepsilon_{t\tau}$ can be interpreted as the elasticity of the expected spot price at the delivery date, T, with respect to the date t+ 1 spot price. If the spot price shock is expected to be permanent, this elasticity equals one. If the spot price is mean reverting, in the sense that a portion of the price shock is expected to be reversed by the delivery date T, then $\varepsilon_{t\tau}$ will be less than one. Though $\varepsilon_{t\tau}$ is assumed to be nonstochastic, it may vary depending on the remaining time until contract expiration, τ.

To link the Samuelson hypothesis to mean reversion in the equilibrium spot price, we apply equation (1) to spot prices at dates t and t+ 1, and use the definitions of u_t and s_t to obtain:

(7) $\ln[E_{t+1}(P_T)/E_t(P_T)] = u_t + \Delta s_t \tau,$

which can be combined with the definition of $\varepsilon_{t\tau}$ to yield:

(8) $\Delta s_t = u_t[\epsilon_{t\tau} - 1]/\tau.$

Expression (8) shows that the cost of carry, s_t, is unchanged in equilibrium if and only if the spot price shock is expected to be permanent ($\varepsilon_{t\tau} = 1$). If the spot price is mean reverting instead ($\varepsilon_{t\tau} < 1$), then the cost of carry must in equilibrium adjust in the opposite direction of the spot price shock. Equation (8) reflects the intuition that, to induce the holding of inventory when prices are mean reverting, the predictable capital loss (gain) in the wake of a positive (negative) price shock must be offset by changes in proportional carrying costs if markets are to clear.

Fama and French (1988a) and Poterba and Summers (1988) show that time variation in the risk premium component of expected returns can induce mean reversion in spot prices. In terms of our analysis, if π were time varying in equation (1), then equation (8) would be restated as $\Delta s_t = u_t[\varepsilon_{t\tau} - 1]/\tau - \Delta\pi_t$, where $\Delta\pi_t \equiv \pi_{t+1} - \pi_t$. This expression shows that mean reversion in equilibrium spot prices could be accommodated by (or caused by) offsetting changes in the risk premium π_t, rather than the cost of carrying inventory, s_t. However, we show in Section A that the Samuelson hypothesis relies on time variation in the cost of carry, s_t. Hence, mean reversion in spot prices that is associated exclusively with variation in the risk premia will not induce the negative comovement between the futures term slope and unexpected returns that is required for the success of the hypothesis. For this reason, we continue to focus on mean reversion associated with shifts in the cost of carry.

Substituting (8) into (5) gives:

(9) $\Delta f_t = \pi + u_t\epsilon_{t\tau}$.

Expression (9) states that the change in the futures price is the ex ante risk premium π, plus the current spot market shock u_t weighted by the elasticity of the expected future spot price with respect to the spot shock. The lower this elasticity, i.e., the greater the proportion of the shock that is expected to be reversed by the delivery date, the smaller the change in the futures price. From (9), it follows that (6) can be restated as:

(10) $VAR(\Delta f_t) = VAR(u_t)(\epsilon_{t\tau})^2$.

Equation (1 O) formalizes relations between the variance of futures price changes and the time-series behavior of spot prices at a general level. The key assumptions invoked to obtain (9) are that the spot asset offers a competitive return (equation (l)) and that the no-arbitrage cost-of-carry relation (2) links contemporaneous spot and futures prices. An immediate implication of (9) is that if $\varepsilon_{t\tau} = 1$, so that all shocks to spot prices are expected to be permanent, (e.g. if spot prices follow a random walk), then the futures price change variance tracks the variance of spot return shocks, regardless of time to delivery. In this scenario, each futures contract's volatility can increase near maturity only if spot return variances increase near each futures delivery date. As noted above, this would imply a saw-toothed volatility pattern over the life of each contract.

If the Samuelson hypothesis does not rely on systematic shifts in the variance of spot shocks near maturity, it must rely on variation in the elasticity of expected future spot prices with respect to current spot shocks. Specifically, since

(11) $d(VAR(\Delta f_t))/d\tau = 2\epsilon_{t\tau}VAR(u_t)(d\epsilon_{t\tau}/d\tau)$,

the volatility of futures price changes depends on time to expiration, τ, if $\varepsilon_{t\tau}$ depends on τ. The Samuelson hypothesis implies that the right side of (11) is negative. Excepting the unlikely case where $\varepsilon_{t\tau}$ is negative (the expected future spot price change is in the opposite direction as the current spot price shock), the Samuelson hypothesis requires $d\varepsilon_{t\tau}/d\tau < 0$, i.e. that revisions in the expected delivery date spot price due to a current shock are smaller for more distant delivery dates.

As an example, consider a spot price process where $\ln[E_{t+1}(P_T)/E_t(P_T)] = u_t e^{a\tau}$. If $a = 0$ the shock u_t is expected to be permanent, while if $a < 0$ the shock is expected to be reversed at a continuous rate equal to a% of the remaining shock. In this example, $\varepsilon_{t\tau} = e^{a\tau}$, $VAR(\Delta f_t) = VAR(u_t)e^{2a\tau}$, and $d(VAR(\Delta f_t))/d\tau = 2aVAR(u_t)e^{2a\tau}$. If $a = 0$ then the variance of futures price changes is unrelated to time to contract maturity, while if $a < 0$ the variance of futures price changes declines with time to maturity and the Samuelson hypothesis is supported. This example illustrates the key role of mean reverting spot prices for the success of the Samuelson hypothesis.

More generally, the variance of futures price changes will increase as the contract delivery date nears during the time horizon over which spot price shocks are reversed. Consider an alternate example in which spot price shocks are expected to be gradually but completely reversed by date A, with expected spot prices beyond date A unaffected by the current shock. This example might be applicable to agricultural commodity markets if current shocks do not affect expected prices beyond the next harvest. Then, the Samuelson hypothesis will be supported for contracts maturing before date A, but since $\varepsilon_{t\tau}$ converges to a constant of zero after date A, it will not be supported for contracts maturing beyond date A.

Our analysis implies that the Samuelson hypothesis will be supported in markets where prices are mean reverting, for delivery dates as distant as the time required for the reversion of spot price shocks to be completed. From (8), these will also be markets where negative covariation between price changes and changes in the futures term slope is observed. The most plausible scenario by which these conditions can be met arises from positive covariation between prices and the convenience yields given off by real assets. Fama and French (1988b) argue that reductions in real asset inventories around business cycle peaks are associated with both increased convenience yields and temporary increases in spot prices. Positive covariation between prices and convenience yields can also arise due to seasonality in production or consumption. Thin inventories prior to harvests, for example, would be associated with temporarily high prices and large convenience yields. Temporary increases (decreases) in convenience yields at times of price peaks (valleys) offset the forecastable capital loss (gain) associated with mean reverting prices, so that agents are willing to hold inventory. In contrast, analogous arguments for financial assets cannot easily be made, since it is difficult to postulate either substantial time-series variation in financial asset inventory or the existence of convenience yields.

To summarize, the Samuelson hypothesis requires either systematic increases in spot return volatility near each futures expiration date or negative covariation between spot returns and the slope of the futures term structure. We argue that the former condition is implausible given that futures contracts mature throughout the year. We focus on the second condition, and show that it will be met in markets where equilibrium spot prices are mean reverting. The hypothesis should not be expected to hold in markets where spot prices follow a random walk, or in markets where spot prices contain a mean reverting component attributable to time variation in risk premia. We argue that the required conditions are much more likely to be met in markets for real assets, especially those where convenience yields display substantial intertemporal variation, than in markets for financial assets. The remainder of this study provides empirical evidence regarding the validity of these predictions.

II. Data Sources and Description

We test our cross-sectional predictions regarding the Samuelson hypothesis using data from agricultural, crude oil, metals, and financial futures. The sample is that employed by Bessembinder, Coughenour, Seguin, and Smeller (1995), and is particularly well suited for testing the validity of our cross-sectional predictions because they report substantial cross-market variation in estimates of the covariation between changes in the futures term slope and price changes. For crude oil and agricultural (wheat, orange juice, live cattle, world sugar, and domestic sugar) markets they report estimates of this covariance that are uniformly negative, significant, and large in absolute magnitude. Their estimates of the covariance between price changes and futures term slope changes for metals (gold, silver, and platinum) are negative and statistically significant, but are orders of magnitude smaller than the estimates for agricultural and crude oil. For equities and Treasury bonds their estimates of the covariance between changes in asset prices and changes in the futures term slope are indistinguishable from zero. Based on these estimates and our analysis presented in Section I, we anticipate that the Samuelson hypothesis will be strongly supported for futures on crude oil and agricultural commodities, that the hypothesis will be supported to a lesser extent for metals, but that the hypothesis will not be supported for futures on Treasury bonds or the S&P 500 index.

We measure daily futures return volatility as the absolute value of daily futures returns, $100 * |R_t|$, where R_t is the continuously compounded percentage change in settlement price since the prior day.[5] Panel A of Table 1 reports average daily volatility by market, as well as numbers of observations.

Before examining volatility in a regression framework, we identify the order of integration of the volatility series, which is important to avoiding misspecifications that could lead to spurious results. Panel B of Table 1 reports results of Dickey and Fuller (1979) unit root tests applied to the time series of nearby contract volatility. Similar results are obtained for more distant contracts. We regress the one day change in nearby volatility on the prior day level of volatility and on five daily lags of the change in volatility. If volatility contains a unit root the change in volatility should be independent of the level. In contrast, if volatility is stationary then the change in volatility should be negatively related to the prior level.

The results indicate that volatility is stationary. In each market the relation between current change in volatility and the prior day level of volatility is negative, and the largest t-statistic on prior day volatility observed in any of the eleven markets is -12.1. Since the critical ($\alpha = .05$) test statistic reported by Fuller (1976) is -2.87, we conclude that volatility in each market is a stationary series that can be analyzed in levels.

[5] We also investigate sensitivity to measuring futures price volatility using a procedure similar to that recommended by Schwert and Seguin (1990). First, futures "returns" are regressed on indicator variables that identify nearby. This purges mean returns as well as any variation in mean returns associated with time to maturity. The absolute value of the resulting zero mean residuals were then scaled by a factor of $(\pi / 2)^{1/2}$. All inferences are robust to employing this method.

HI. Evidence on the Samuelson Hypothesis

A. Average Volatility by Nearby

We first investigate the behavior of futures price change volatility by computing mean volatility for a variable we term "nearby", which indicates relative nearness to contract expiration. For example, nearby one is the contract closest to delivery (except that we exclude prices of contracts within their delivery month), nearby two is the second nearest, and so on. Table 2 reports mean volatility for each contract by nearby. These estimates are obtained by regressing daily volatility measures on indicator variables that identify nearby. We also report results of a formal test that mean volatility is equal across nearby in each market.[6]

For agricultural commodities and crude oil the hypothesis that average volatility is equal across nearbys can be readily rejected, and the point estimates indicate that average volatility tends to increase as the expiration date nears. These results imply strong support for the Samuelson hypothesis. The increases in volatility are generally substantial. For example, contrasting the volatility of the 4th nearest to expiration contract with the nearest to expiration contract indicates that nearby volatility is greater by 39% in domestic sugar, 32% for world sugar, 43% for cattle, 21% for crude oil, 24% for wheat, and 21% for orange juice.[7]

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[6] The structure of this dataset generates some econometric complications. There are two sources of dependence among the observations: prices on a given trading date for different delivery dates are highly correlated, and observations on the futures term slope and implied cash flow yields for each contract are serially autocorrelated. We use Newey-West (1987) standard errors to accommodate these sources of dependence as well as generalized forms of heteroskedasticity. We sort the data by trading date and then by delivery date and, letting K denote the number of nearbys involved, employ K+ 1 lags to compute the Newey-West covariance matrix. This procedure accommodates dependence across the K prices observed on each trading date as well as first order autocorrelation. Joint hypotheses are tested using a Wald statistic: $W = (RB)'(RNR')^{-1}(RB)$, where R is the restriction matrix of rank p, B is the coefficient matrix, and N is the Newey-West matrix. This statistic follows an asymptotic chi-square distribution with p degrees of freedom.

[7] Since the number of contract expirations per year varies across markets, the calendar time associated with changes in nearby varies across markets. For example, crude oil contracts expire each month, implying that nearby =2 is two months from expiration. In contrast the S&P 500 contracts expire quarterly, so nearby =2 includes trading in contracts four to six months to expiration. Contract expiration months are reported at the bottom of Table 2.

Point estimates for metals markets are generally consistent with the predictions of the Samuelson hypothesis, with estimates of mean volatility generally increasing as expiration nears. However, the increases are much less dramatic, with increases in volatility from the 4th nearest to the nearest contract of only 2.8% for silver, 0.6% for gold, and 2.9% in platinum. Also, the hypothesis that volatility is equal across nearbys is not rejected for any of the three metals markets.

Empirical estimates for financial futures provide no support for the Samuelson hypothesis. For both Treasury bonds and the S&P 500 index, point estimates of volatility are virtually identical for near and distant delivery. The hypothesis that volatility is constant across nearbys cannot be rejected for either market, and each p-value exceeds .96.

B. Direct Estimates of Relations between Volatility and Time to Maturity

We next estimate the relation between time to contract maturity and the volatility of futures price changes for each market directly. To do so, we stack all of the volatility observations for a given market and estimate regressions of volatility on the square root of the number of days until the contract expires. We also investigate the sensitivity of test results to the use of the raw number of days to expiration and the natural log of the number of days to expiration, and find that conclusions are wholly unaffected. The dependencies across observations that result from stacking observations from common trading dates are accommodated by use of the Newey-West (1987) covariance matrix.

Panel A of Table 3 reports results of simple regressions of the daily volatility estimates on the square root of the number of days until the contract delivery date. Consistent with the Samuelson hypothesis, estimated coefficients on the time to expiration variable reported in Panel A of Table 3 are negative for every futures market except the S&P 500. However, the magnitude and statistical significance of the coefficient estimates vary cross-sectionally. The largest (absolute) coefficient estimates are obtained for crude oil, world sugar, cattle, wheat, and orange juice. For each of these markets the estimated coefficient on time to expiration differs significantly from zero. In contrast, estimates for Treasury bonds, metals, domestic sugar, and the S&P 500 are statistically indistinguishable from zero.

C. The Effect of Controlling for Variation in Information Flow

We next extend the analysis of each market to also include a proxy the volatility of returns to the underlying asset. Controlling for spot return volatility distinguishes between our presentation of the Samuelson hypothesis and reinterpretations such as that suggested by Anderson and Danthine (1983) that link the hypothesis to changes in the rate of information flow. Lacking reliable spot price data for several of these markets, we follow Fama and French (1988b) in using near futures prices as a proxy for spot prices. This limits the analysis to use of volatility for nearbys of two and greater as the dependent variable. Results of this specification are reported on Panel B of Table 3. In Panel C of Table 3 we report results obtained when monthly intercept indicators, which accommodate potential seasonalities in information flow, are also included as regressors. If the present analysis, which indicates that mean reverting spot prices are the key to the success of the Samuelson hypothesis, is correct, then we should observe negative coefficient estimates on the days to expiration variable in those markets where prices are mean reverting, independent of whether the information flow variables are included in the regressions.

Estimated coefficients on the spot volatility proxies reported on Panel B of Table 3 are uniformly positive, as might be expected, and each is statistically significant. Importantly, the addition of the spot volatility proxy has little effect on point estimates of parameters associated with time to expiration. Indeed, the main discernable effect of including the spot volatility proxy is to decrease the standard errors associated with time-to-maturity estimates. As a consequence, the negative coefficient estimates on days to expiration in the three metals markets are statistically significant in this specification. Coefficient estimates on time to expiration remain insignificant for equity index and Treasury bond futures.

The chi-square statistics reported on Panel C of Table 3 indicate that the hypothesis that the eleven estimated coefficients on the monthly indicators jointly equal zero can be rejected for each of the eleven markets. This indicates significant seasonalities in volatility for every market. As might be expected, these seasonalities in volatility are partially attributable to harvest cycles; for example, intercepts for wheat futures are highest in the late spring and early summer. Most importantly, however, estimated coefficients on the square root of the number of days to contract expiration are little altered relative to those reported on Panel A or B. We conclude that, though each market is characterized by seasonal patterns in volatility, inference regarding whether or not the Samuelson hypothesis holds is robust to these seasonalities.

These empirical results affirm the predictive power of this analysis. Whether futures prices in a particular market behave in accordance with the Samuelson hypothesis is governed by the relation between spot price shocks and the slope of the futures term structure. The Samuelson hypothesis is strongly supported in markets for agricultural and crude oil, where strong negative relations between prices and futures term slopes are also observed, indicating substantial mean reversion in spot prices. In metals markets, where the degree of spot price mean reversion is less, the Samuelson hypothesis is supported, but less strongly. In financial markets, which are characterized by the absence of a significant relation between prices and the futures term slope, indicating the absence of significant mean reversion in prices, the Samuelson hypothesis is not supported. These empirical results are robust to the inclusion of variables that proxy for the rate of information flow.

IV. Conclusions

The Samuelson hypothesis predicts increasing futures price change volatility as contract expiration nears. In markets where the hypothesis holds it is necessary to estimate a term structure of futures volatilities, in which volatility depends on time to contract expiration, in order to accurately value options or related derivatives on futures.

This study provides clarification of the conditions under which the Samuelson hypothesis should be expected to hold. We show that the hypothesis requires either systematic increases in spot return volatility near each futures contract expiration or negative covariation between spot returns and changes in the slope of the futures term structure. We argue that the former condition is implausible, and show that the second condition depends on the time series behavior of the associated spot price. In particular, negative covariation between spot prices and changes in the futures term slope requires that spot prices changes include a temporary component. In markets where the spot price follows a random walk, or where the price is mean reverting but the reversion reflects time variation in risk premia only, the hypothesis should not be expected to hold. The hypothesis will be supported in markets where spot prices are mean reverting and the reversion is associated with variation in the cost of carrying inventory. Intuition suggests, and the empirical evidence confirms, that such negative covariation will be observed primarily in markets where real service flows (convenience yields) exist and display substantial intertemporal variation. Since financial assets do not provide real service flows, this analysis implies that the Samuelson hypothesis is unlikely to hold in financial futures markets.

We test our predictions using data from eleven futures markets. As predicted, the Samuelson hypothesis is strongly supported in markets for agricultural and crude oil, where strong negative relations between prices and futures term slopes are also observed, indicating the presence of a large mean reverting component in spot prices. In metals markets, where the degree of spot price mean reversion is less, the Samuelson hypothesis is supported, but less strongly. In financial markets, which are characterized by the absence of a significant relation between prices and the futures term slope, the Samuelson hypothesis is not supported. These empirical results are robust to the inclusion of variables that proxy for the rate of information flow. On balance, this evidence provides strong support for the implication that the Samuelson hypothesis should be expected to hold only in markets where spot price changes contain a significant temporary component.

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Table 1: Descriptive and Diagnostic Information for Futures Return Volatility.

The sample is comprised of daily observations over the interval January 1982 to December 1991, except for the S&P 500 (sample starts April, 1982), crude oil (sample starts March 1983) and domestic sugar (sample starts September 1985). Daily volatility is measured as the absolute value of the daily futures return (continuously compounded percent change in settlement price since the prior day). Panel A reports the sample size and average daily volatility for each contract. The results of Dickey-Fuller test for a unit root in volatility are reported in Panel B. Standard errors are reported in parentheses.

Panel A: Descriptive Statistics

	Crude Oil	Silver	Gold	Platinum	Wheat	Orange Juice	Domestic Sugar	World Sugar	Cattle	Treasury Bonds	S&P 500
Observations	17,045	17,171	19,859	11,668	12,420	16,502	9,219	12,302	14,695	19,809	9,342
Average Daily Volatility	1.5786	1.6962	1.0297	1.6524	1.0248	1.0256	0.2421	2.2145	0.7075	0.7190	1.0233

Panel B: Coefficients from regression of the change in volatility ($\Delta\sigma_t$) on the prior day's level of volatility (σ_{t-1}) and five lags of change in volatility ($\Delta\sigma_{t-i}$).

Independent Variables	Crude Oil	Silver	Gold	Platinum	Wheat	Orange Juice	Domestic Sugar	World Sugar	Cattle	Treasury BQ!MiS	S&MQQ
σ_{t-1}	-.3735	-.4684	-.5309	-.4003	-.6877	-.5186	-.4246	-.6466	-.6047	-.5455	-.4491
	(.0287)	(.0309)	(.0287)	(.0277)	(.0273)	(.0283)	(.0352)	(.0313)	(.0281)	(.0376)	(,0340)
$\Delta\sigma_{t-1}$	-.4266	-.4094	-.3597	-.4673	-.2237	-.3553	-.3895	-.2968	-.3513	-.4359	-.4281
	(.0307)	(.0314)	(.0289)	(.0287)	(.0260)	(.0279)	(.0365)	(.0298)	(.0267)	(.0369)	(.0342)
$\Delta\sigma_{t-2}$	-.3141	-.3615	-.2921	-.3831	-.1949	-.2492	-.1325	-.2671	-.2809	-.3885	-.3202
	(.0300)	(.0302)	(.0275)	(.0279)	(.0238)	(.0267)	(.0355)	(.0277)	(.0248)	(.0353)	(.0329)
$\Delta\sigma_{t-3}$	-.2123	-.2779	-.2008	-.2651	-.1494	-.1629	-.1886	-.2014	-.2011	-.2883	-.2041
	(.0284)	(.0279)	(.0254)	(.0263)	(.0215)	(.0249)	(.0327)	(.0248)	(.0233)	(.0322)	(.0308)
$\Delta\sigma_{t-4}$	-.1519	-.1804	-.1164	-.1954	-.1103	-.1025	-.1192	-.1449	-.1294	-.1890	-.1883
	(.0254)	(.0241)	(.0221)	(.0230)	(.0184)	(.0218)	(.0304)	(.0211)	(.0188)	(.0271)	(.0268)
$\Delta\sigma_{t-5}$	-.0317	-.0641	-.0352	-.0711	-.0574	-.0628	-.0231	-.0993	-.0664	-.0641	-.0579
	(.0205)	(.0186)	(.0172)	(.0177)	(.0143)	(.0167)	(.0240)	(.0158)	(.0135)	(.0196)	(.0203)
Adj. R^2	.4122	.4648	.4818	.4447	.5669	.4668	.4579	.5389	.6326	.5041	.4414

Table 2: Mean Futures Return Volatility By Nearby.

Each entry is the mean of daily volatility estimates, computed as the absolute value of the daily futures return (continuously compounded percent change in settlement price since the prior day), by nearby. Nearby = 1 denotes use of prices for the closest to maturity contract, Nearby = 2 the second nearest to maturity, etc. Joint hypothesis tests employ a Wald statistic, and use the covariance matrix described by Newey and West (1 987).

Nearby	Crude Oil[a]	Silver[b]	Gold[b]	Platinum[c]	Wheat[d]	Orange Juice[e]	Domestic Sugar[e]	World Sugar[f]	Cattle[b]	Treasury Bonds[g]	S&P 500[g]
1	1.853	1.740	1.034	1.675	1.183	1.199	0.308	2.641	0.903	0.720	1.012
2	1.682	1.723	1.032	1.662	1.071	1.109	0.238	2.370	0.836	0.721	1.020
3	1.600	1.705	.031	1.644	0.993	1.041	0.220	2.138	0.703	0.718	1.023
4	1.536	1.693	.028	1.627	0.952	0.992	0.221	2.003	0.631	0.717	1.033
5	1.493	1.667	.024	1.656	0.925	0.951	0.236	1.904	0.595	0.716	
6	1.463	1.654	.021			0.908	0.230		0.562	0.716	
7	1,473	1.716	1.018			0.949				0.719	
8	1.519		1.047							0.721	
Test: Equal Across Nearbys	52.03 (.000)	4.81 (.568)	0.92 (.996)	1.30 (.862)	104.82 (.000)	99.49 (.000)	33.09 (.000)	200.74 (.000)	464.95 (.000)	(.999)	0.27 (.967)

‘Delivery months are 1,2,3,4,5,6,7,8,9, 10, 11, 12
[b] Delivery months are 2,4,6,8, 10, **12**
[c] Delivery months are 1,4,7, 10
[d] Delivery months are 3,5,7,9, 12
[e] Delivery months are 1,3,5,7,9, 11
‘Delivery months are 3,5,7, 10
[g] Delivery months are 3, 6, 9, 12

Table 3: Estimates of Relations Between Futures Volatility and Days to Contract Expiration (DTE).

Daily volatility estimates are regressed on the square root of days to expiration (DTE) in Panel A, and also on spot return volatility in Panel B. Panel C also includes monthly indicators. The standard error of the DTE coefficient in parentheses and a joint hypothesis test for equal volatility across calendar months are reported in Panel C. The standard errors and the joint hypothesis tests employ the covariance matrix described by Newey and West (1987).

Panel A: Regression of Daily Volatility on Days to Expiration

	Crude Oil	Silver	Gold	Platinum	Wheat	Orange Juice	Domestic Sugar	World Sugar	Cattle	Treasury Bonds	S&P 500
Intercept	1.51	1.41	0.84	1.35	0.95	1.03	0.17	2.19	0.88	0.57	1.48
$\sqrt{DTE}$	-.0255	-.0052	-.0019	-.0029	-.0116	-.0164	-.0008	-.0326	-.0251	-.0002	,0108
Standard Error	(.0093)	(.0049)	(.0028)	(.0062)	(.0040)	(,0037)	(.0016)	(.0051)	(.0022)	(.001 1)	(.0143)

Panel B: Regression of Daily Volatility on Days to Expiration and Spot Volatility

Independent Variables	Crude Oil	Silver	Gold	Platinum	Wheat	Orange Juice	Domestic Sugar	World Sugar	Cattle	Treasury Bonds	S&P 500
Intercept	0.55	0.12	0.02	0.09	0.24	0.47	0.09	1.01	0.48	0.01	0.61
Spot Volatility	.6268	.9480	.9815	.9558	.7305	.5667	.2941	.6162	.5667	.9727	.9872
$\sqrt{DTE}$	-.0225	-.0069	-.0012	-.0045	-.0100	-.0146	-.0015	-.0395	-.0259	-.0002	-.0017
Standard Error	(.0033)	(.0006)	(.0002)	(.0006)	(.0018)	(.00 19)	(.0013)	(.0029)	(.0013)	(.0002)	(.0012)

Panel C: Regression of Daily Volatility on Days to Expiration , Spot Volatility, and Monthly Indicators

Independent Variables	Crude Oil	Silver	Gold	Platinum	Wheat	Orange Juice	Domestic Sugar	World Sugar	Cattle	Treasury Bonds	S&P 500
Intercept	0.57	0.14	0.03	0.11	0.25	0.85	0.13	1.09	0.48	0.02	0.06
FEB	-.0059	-.0344	-.0120	-.0189	-.0089	-.2490	-.0711	-.1254	.0467	-.0041	-.0045
MAR	.0621	-.0313	-.0039	-.0029	.0167	-.3798	-.0589	-.0805	-.0181	-.0053	-.0036
APR	-.1204	-.0223	-.0029	.0082	-.0209	-.3897	-.0468	-.0952	.0161	-.0057	-.0064
MAY	-.1683	-.0515	-.0013	-.0094	.1919	-.04697	-.0385	-.0851	.0164	.0025	-.0329
JUN	-.1065	-.0252	-.0038	-.0282	.0958	-.3638	-.0890	.0397	.0766	-.0004	.0262
JUL	-.0249	-.0111	-.0181	-.0294	.1985	-.4778	-.0853	-.0125	.0353	-.0135	-.0186
AUG	-.0234	-.0337	-.0184	-.0254	.0742	-.4645	-.0642	-.0446	-.0086	-.0062	-.0271
SEP	.0311	-.0416	-.0188	-.0192	.1474	-.4682	-.0146	-.0869	.0401	.0086	.0249
OCT	.0906	-.0270	-.0058	-.0093	.0299	-.3027	-.0296	.1503	.0039	-.0141	-.0455
NOV	-.0306	-.0160	-.0022	-.0061	-.0248	-.4490	-.0054	.0847	.0025	-.0038	-.0086
DEC	.0973	-.0311	-.0011	.0004	.0358	-.2439	.0000	.0059	-.0253	.0022	.0234
Spot Volatility	.6247	.9481	.9818	.9562	.7243	.5407	.2879	.6178	.5652	.9730	.9879
$\sqrt{DTE}$	-.0225	-.0068	-.0012	-.0045	-.0150	-.0143	-.0018	-.0432	-.0261	-.0003	-.0010
Standard Error	(.0032)	(.0006)	(.0002)	(.0006)	(.0018)	(.0017)	(.0013)	(.0029)	(.0014)	(.0002)	(.0012)
Test: All Monthly Indicators=0 χ^2 (p-value)	110.43 (.000)	41.05 (.000)	69.41 (.000)	42.85 (.000)	223.58 (.000)	196.97 (.000)	70.87 (.000)	83.96 (.000)	45.00 (.000)	37.82 (.000)	56.03 (.000)